INTERNATIONAL MONETARY FUND

Optimal Policy for Financial Market Tokenization

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WP/25/185

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2025 SEP



IMF Working Paper Research Department

Optimal Policy for Financial Market Tokenization Prepared by Itai Agur and Alexander Copestake *

Authorized for distribution by Maria Soledad Martinez Peria September 2025

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ABSTRACT: Competing broker initiatives to "tokenize" financial assets—i.e., represent them on programmable platforms—promise efficiency gains but raise concerns about market fragmentation. Policymakers in several countries are considering supporting such platforms or mandating their interoperability. We provide the first formal framework for analyzing optimal policy in this context. Brokers with heterogeneous market power compete to attract investors and execute their trades intra-broker or on a legacy platform. Coalitions of brokers can invest in creating a tokenized market with faster, cheaper inter-broker settlement. Partial coalitions divert trades away from excluded competitors, leading to equilibrium coalition structures that can feature excessive investment or insufficient tokenization. Neither public-private cost-sharing nor interoperability mandates are sufficient to achieve the social optimum when used alone, but their combination is. These results withstand incorporating an open-access ledger (e.g., a public blockchain).

JEL Classification Numbers:	C72, D85, G24, O33.
Keywords:	Tokenization; Interoperability; Intermediation; Trading platforms; Coali-tion formation.
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^{*} We are grateful to Charles Kahn (discussant), Thorsten Koeppl (discussant), Giovanni Dell'Ariccia, Michael Junho Lee, Filippo Mezzanotti, and audiences at the 2025 CEBRA Annual Meeting, IFABS 2025, the 2025 International Conference on Payments and Securities Settlement, 2025 Summer Workshop on Money, Banking, Payments and Finance, the 2025 IMF Annual Macro-Financial Research Conference and an internal IMF seminar for helpful comments.

WORKING PAPERS

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1 Introduction

The financial press is abuzz with talk of tokenizing financial assets, which is the creation of assets or representations of assets on a shared and programmable ledger (Agur et al., 2025b; Aldasoro et al., 2023). Programmability means that self-executing contracts can be written on the ledger, which for instance enables the simultaneous and instantaneous exchange of a seller's asset and a buyer's payment. While the measurement of tokenization's potential impact is in its infancy, the first estimates indicate that the efficiency gains—such as savings on settlement cost and time from automating some of the roles of specialized intermediaries like registrars and clearing houses—could be non-negligible. For instance, J.P. Morgan (2023) estimates that the continuous reinvestment of cash that is currently locked in during settlement on US financial markets reduces portfolio management costs by 22 percent. Several major brokers, including BlackRock, Goldman Sachs and JP Morgan, have announced tokenization initiatives, as have coalitions of brokers (e.g., Canton Tokenization Network; Regulated Settlement Network), leading to estimates of up to 16 trillion US dollars of tokenized financial assets by 2030.

Responding to these developments, policy institutions are beginning to take an active role in the formation of tokenized financial markets. On one hand, they envision potential efficiency gains; on the other, the proliferation of competing private ledger initiatives raises concerns of market fragmentation. Policymakers' approaches range from ensuring the interoperability of any privately created ledgers, as is the case in Singapore and under consideration in the UK, to public-private partnerships where the policymaker can take a leading role in the creation and operation of the new market infrastructure, as seen in Brazil and

¹For a detailed discussion of the features of programmability, see Lavayssière and Zhang (2024).

²Moreover, in matched samples of tokenized and conventional bonds, Leung et al. (2023) find that underwriting fees and bid-ask spreads on tokenized bonds are, respectively 25.8 and 5.3 percent lower, and Aldasoro et al. (2025) find that mean bid-ask spreads are 37 percent lower while issuance costs are comparable. For a matched sample of tokenized and traditional asset-backed securities in China, Liu et al. (2023) estimate that tokenization on average lowers yields by 25 basis points.

the multinational Project Agorá.³

The involvement of policymakers in the formation of tokenized markets raises several questions. If preventing market fragmentation is the main justification for policy, what are the economic forces driving brokers toward such fragmentation and why is this socially costly? What are the tradeoffs that policymakers face as they determine whether and to what extent to become involved in tokenized market formation? If policy intervention is needed, would regulation, like mandating interoperability, suffice? Or is there a need for the public sector to go further, such as by sharing costs in a public-private partnership to create a tokenized financial market? To help analyze these questions, we develop the first model of optimal policy for the formation of tokenized markets.

We model an endowment economy containing retail investors that need brokers to match them to each other to trade. Investors are initially assigned to one broker and face switching costs to move to a different broker, which gives brokers market power. The first key feature we aim to capture is heterogeneity in this market power. The second is the possibility of fragmentation of inter-broker trade. We study the simplest structure that delivers both of these features: three brokers (the minimum number required for fragmentation to be possible) and six investors, distributed unevenly (such that each broker has a different positive number of initial clients, and hence different market power). Specifically, Broker 1 has one investor as an initial client, Broker 2 has two, and Broker 3 has three.

Except for their initial broker assignment, the investors are ex-ante identical. However, after having chosen a broker, a shock sorts the investors into two different groups, of potentially unequal size, that each desire to trade with the other. This sorting represents, for example, a liquidity shock that turns the affected subset of investors into sellers of an asset while the others take the buy side.

Brokers can use up to three trading modalities, which clear sequentially, to match their

³Public sector involvement could also take the form of providing of a new means of payment for use on the tokenized market, as contemplated in Australia, Brazil and Switzerland. See Section 7 for further discussion.

⁴On the importance of this feature in brokerage markets see, e.g., Duffie (2022) and Dugast et al. (2022).

investors. In the first trading round, each broker clears any feasible within-broker matches of clients with complementary types, at a per-match cost normalized to zero. In the last trading round, brokers clear outstanding trades on a pre-existing inter-broker market that all brokers can access and that operates with legacy technology involving positive transaction costs for brokers. In between these two rounds, some or all brokers can also settle matches on a new inter-broker market that uses improved infrastructure, if they agreed to form such a market at the start of the game. We refer to this as the 'tokenized market' and, in line with stylized facts, this market has an advantage in both speed, as it clears earlier, and cost, with the transaction cost also normalized to zero. However, participating brokers incur an initial cost to set up the tokenized market.

Brokers earn revenue from transaction fees and can price discriminate between their initial clientele and investors switching from other brokers, in line with the sweeteners that brokers offer new clients in practice. Toward each broker's initial client base, brokers engage in Bertrand competition subject to differentiation on marginal costs (depending on the share of trades that a broker expects to process on the legacy market) and investor matching probabilities (from differentiated access to trading modalities).⁵

At the start of the game, brokers negotiate about forming a tokenized market, foreseeing the implications for their profits through the channels above. The brokers engage in a coalition formation game that results in no coalition, a partial coalition of any two brokers, or the grand coalition of three brokers. An equilibrium is found when none wish to individually or jointly deviate to a different coalition.

Solving for the equilibrium of the game, we find that the grand coalition never forms. When brokers find a tokenized market too costly to set up, no coalition forms. In contrast, when brokers find that the benefits of a tokenized market justify its cost, a partial coalition forms. The underlying force is trade diversion. When one broker is excluded from the tok-

⁵Rather than resolving which investors come to trade through heterogeneous asset valuations and bidding, here all investors looking to take a given side of the transaction place the same value on the potential trade. Investors' heterogeneous access to trading technologies via brokers then determines their likelihood of a match. This centers our model on the role of endogenous market infrastructure while retaining tractability.

enized market, the included brokers expect to process a larger share of all investor matches. This outweighs the desire to minimize the cost of the average trade, which is zero when the legacy market is retired under the grand coalition. In equilibrium, the largest broker (Broker 3) and the smallest (Broker 1) band together to divert trade from the third (Broker 2).

A partial coalition is socially suboptimal. Identifying welfare as the sum of investor utility and broker profit, we find that welfare is highest under either the grand coalition or no coalition, depending on the setup cost and efficiency gains of the tokenized market. Intuitively, if the setup cost of the tokenized market is to be incurred, all brokers should be included on this market to eliminate costly legacy market transactions. When partial coalitions form in equilibrium, they instead involve either excessive investment or insufficient tokenization. Excessive investment occurs if a tokenized market socially costs more than it is worth, but the private benefit of trade diversion nonetheless leads to its formation through a partial coalition. Insufficient tokenization results when forming the tokenized market maximizes welfare, but the private benefit of trade diversion leads two brokers to exclude the third from the market instead of accepting the grand coalition.

Turning to policy, we consider whether an interoperability mandate can attain the social optimum. We incorporate this mandate by giving any initially excluded broker the option to subsequently join the tokenized market. This makes trade diversion infeasible in equilibrium and counteracts excessive investment. However, this does not imply that a three-broker tokenized market forms whenever it is socially optimal. For a small enough setup cost, all brokers prefer the grand to no coalition and interoperability then resolves insufficient tokenization. But when the setup cost is larger, the least supportive broker vetoes the grand coalition, so no tokenized market forms at all. Interoperability here leads to underinvestment for two reasons. First, achieving unanimity on a grand coalition is vulnerable to heterogeneous broker incentives. Second, part of tokenization's efficiency gains accrue to investors through lower fees in equilibrium.

Escaping this underinvestment problem requires a second instrument. We let the poli-

cymaker tax investors lump-sum to cover part of the cost of tokenized market formation, in the vein of public-private tokenization initiatives. When used alone, this tool never improves the equilibrium coalition structure. But, in our model, the combination of cost sharing and an interoperability mandate *can* achieve the socially optimal outcome in all cases.

We explore two extensions to the baseline model. First, we expand brokers' bargaining space. In our baseline tokenized market formation game, brokers can only choose whether or not to form coalitions and any setup costs are shared equally among participating brokers. We relax this assumption through a transferrable utility game among the brokers, where unbounded transfers to (dis)incentivize the formation of any coalition are allowed. In this setting, the maximization of aggregate broker profit drives equilibrium coalition formation. Insufficient tokenization can nevertheless result because tokenization benefits retail investors through lower fees in equilibrium, which brokers fail to internalize. According to our model, a role for policy—specifically, fiscal transfers between investors and brokers—thus remains.

Second, we extend to broker trade on an independent tokenized platform that comes without a setup cost and is open for all brokers to join. That is, in addition, to creating separate proprietary tokenized markets, brokers have the option to join a tokenized market that operates on pre-existing infrastructure, such as a public blockchain.⁶ We find that even in this setting policy remains necessary to ensure the optimal formation of tokenized markets.

The existing theoretical literature on tokenized markets has focused on tradeoffs inherent to the operation of such markets rather than the process of their formation. In Chiu and Koeppl (2019), the validation of transactions on a (decentralized) tokenized market relies on a consensus mechanism that more credibly addresses settlement risk when more miners join, which however depends on rewards that increase as congestion rises and the speed of settlement declines. Lee et al. (2024a,b) highlight a tradeoff between the benefits of simulta-

⁶This extension is inspired by the fact that several brokers, such as BlackRock, Fidelity, Franklin Templeton and Robinhood, are offering tokenized securities on public blockchains, including Ethereum, although these initiatives at present are best described as building separate tokenized markets (with selective access for investors and other financial institutions) as layers on top of a public blockchain.

neous settlement and the potential costs of a liquidity hold-up problem from instantaneous settlement. The latter originates in the requirement for the seller to own the asset at the time of trade, which reveals information that can be exploited by the buyer. These papers analyze the optimal design of a tokenized market in view of this tradeoff, where Lee et al. (2024a) center on the joint design of settlement and trading systems and Lee et al. (2024b) focus on program execution and transaction verifiability.⁷

Our prime contribution is providing a framework to analyze the policies for financial market tokenization that are under active consideration. Our paper aims for the most general model capable of fulfilling this objective. As a result, our framework can also speak to other settings where (some) participants on an established market choose whether to adopt a different trading technology. There is a long history to such choices. For instance, Banner (1998) describes the 1792 Buttonwood Agreement that founded the New York Stock Exchange (NYSE) as the formation of an efficient but exclusionary cartel for trade among a small set of brokers. Nowadays, with the NYSE as the shared-access venue for US equity brokers, fragmentation instead takes the form of exclusive side-exchanges. These include formal exchanges with limited broker membership that lay claim to more efficient trading technology than the NYSE, such as MEMX, but also so-called 'dark pools' that arguably use less efficient technologies than the NYSE but nevertheless processed, respectively, 16 and 13 percent of US equity trading volume and value in 2024 (Brugler and Comerton-Forde, 2025; Halim et al., 2025). Our model predicts that, if the fixed cost of adopting the alternate

⁷Programmable tokens have also been considered in other contexts than financial asset trade, including for domestic and cross-border payments (Adrian et al., 2023, 2022; Chiu and Monnet, 2025; Kahn and van Oordt, 2022), trade finance (Cong and He, 2019), credit contract enforcement (Brunnermeier and Payne, 2023), bank runs (Georgiadis-Harris et al., 2024), synthetic financial products (Rostek and Yoon, 2024), platform governance (Abadi and Brunnermeier, 2024; Auer et al., 2025; Cong et al., 2025; Li and Mann, 2025; Reuter, 2024; Sockin and Xiong, 2023), monopolistic product pricing (Brzustowski et al., 2023), truthful disclosure (Garratt and Monnet, 2023), monetary policy (Project Pine, 2025), resolving equilibrium multiplicity (Aronoff and Townsend, 2025), and social planner automation (Townsend and Zhang, 2023).

⁸Dark pools generally have over-the-counter (OTC) trading structures while central stock exchanges trade with a limit-order-book (LOB). Trading OTC can benefit some agents, including brokers, but trading LOB tends to lower bid-ask spreads for investors at large (Benos et al., 2022; Bessembinder et al., 2020; Degryse et al., 2014; Duffie, 2022; Rostek and Yoon, 2025; Weill, 2020). For example, Israeli corporate bonds traded LOB have markedly smaller bid-ask spreads than US corporate bonds traded OTC (Abudy and Wohl, 2018).

trading technology is not too large, partial coalitions of brokers adopt it even when it is less efficient than the central market's technology.

Our paper thus also relates to the literature on the endogenous formation of financial market trading structures. Dugast et al. (2022) are closest to us within this literature, modeling intermediaries of heterogeneous trading capacity that choose between centralized and OTC markets. Small intermediaries prefer the OTC market due to the risk sharing that large brokers can profitably provide to them there, while mid-sized intermediaries opt for the centralized market. This resembles the emergence of a partial coalition between the largest and smallest brokers in our equilibrium. However, in Dugast et al. (2022) neither market enables exclusion, as brokers are free to join either. Brokers thus have two pre-existing, open-access markets with different features to choose from—put differently, the framework is closer to our model with the interoperability policy already in place and the setup of the new market pre-funded.

The remainder of the paper is organized as follows. The next section presents the setup of the model. Section 3 derives the private sector equilibrium. Section 4 assesses welfare, which Section 5 uses to analyze socially optimal policy. Section 6 summarizes the model extensions and Section 7 concludes. Proofs can be found in the Appendix.

⁹Other papers in this literature analyze the emergence of market structure under different forms of heterogeneity, including heterogeneity in investors' asset valuations (Babus and Parlatore, 2022), wealth (Koeppl, 2012), risk preferences (Malamud and Rostek, 2017), search abilities (Lu et al., 2025), and trading intensity (Farboodi et al., 2023; Sambalaibat, 2022; Üslü, 2019), as well as endogenous market entry of investors (Chen and Duffie, 2021; Duffie et al., 2017; Lee and Wang, 2025; Pagano, 1989) and intermediaries (Atkeson et al., 2015; Biais, 1993; Bolton et al., 2016; Chang and Zhang, 2021; Chiu et al., 2020; Farboodi, 2023; Farboodi et al., 2019; Miao, 2006; Rust and Hall, 2003; Wang, 2024).

¹⁰Exclusive coalition formation also distinguishes us from papers on externalities in financial technology adoption (Alvarez et al., 2023; Crouzet et al., 2023; Higgins, 2024; Pagnotta and Philippon, 2018), including papers on interoperability among payment providers (Alok et al., 2024; Bianchi et al., 2023; Bianchi and Rhodes, 2024; Bouvard and Casamatta, 2024; Brunnermeier et al., 2023; Chiu and Wong, 2022; Copestake et al., 2025; Frost et al., 2025).

2 Model

Our model contains two types of agents: investors and brokers. Investors in the model are retail investors, who cannot trade with each other directly. To trade, they must therefore engage the services of a broker. Investors and brokers are risk neutral and fully informed about the structure of the game that we describe below.

2.1 Investors

Investors are born with a money endowment, η , which they can use for payments, while any unused portion enters the investor's utility linearly (e.g., from consumption paid for with the remaining endowment).

Each investor begins the game attached to a particular broker and we refer to these starting attachments as the initial clientele of a broker. There are three brokers, who start with one, two and three initial investors, respectively. Brokers and investors are numbered according to the initial client base of the broker. Specifically, Broker 1 starts with one investor, Investor 11; Broker 2 starts with two investors, Investor 21 and Investor 22; and Broker 3 starts with three investors, Investor 31, Investor 32 and Investor 33. Written more generally, we denote investors by their initial broker l and a within-initial-broker identifier i, such that $li \in \{11, 21, 22, 31, 32, 33\}$.

We assume that an investor can only be a client of one broker, although this need not be its initial broker. An investor can choose to become a client of a different broker, but this involves a switching cost, τ . This is a non-pecuniary inconvenience cost that reflects the substantial time and effort that can be required to transfer assets between brokers.¹¹

Aside from their starting broker attachments, investors are initially homogeneous. At an intermediate stage of the game, investors are exogenously assigned one of two types, a and b, that can each realize a 'gain from trade' by interacting with the other via a broker. These types are a stylized representation of any source of ex-post heterogeneity that generates

 $^{^{11}}$ See, for instance, the discussion on inter-broker asset transfer in Agur et al. (2025b).

differences in the valuations of an asset, and hence a desire to trade. For instance, type b could reflect investors hit by a liquidity shock that makes them willing to sell a future income stream to type a investors at a discount to its held-to-maturity value.¹²

On discovering their type $k \in \{a, b\}$, investors enjoy gains from trade, normalized to one, if they are able to interact with the opposing type through their chosen broker. Broker j charges a fee f_{lj} to facilitate this, as discussed further in Section 2.2.

We assume for simplicity that investors are equally likely to be assigned to type a as to type b.¹³ We denote by P_{kj} the probability that an investor of type k is matched with its opposite type, conditional on choosing to trade through broker j. The expected utility, u_{li}^e , of an investor li choosing broker j is then given by

$$u_{li}^{e} = \eta + \max\{(1 - f_{lj}) \left(\frac{1}{2}P_{aj} + \frac{1}{2}P_{bj}\right), 0\} - \tau_{lj}$$
(1)

where $\tau_{lj} \in \{0, \tau\}$ and $\tau_{lj} = \tau$ if $l \neq j$ (whereas $\tau = 0$ when l = j, which means that the investor stays at its initial broker).¹⁴ The max operator reflects that investors will not place a trade unless the expected net return from doing so is weakly positive.

2.2 Brokers

Brokers earn profits by charging fees to facilitate investors' trades. As noted above, Broker 3 initially has one more investor than Broker 2, who initially has one more investor than Broker 1. This leads to differentiation in market power among them: Broker 3 has the most market power—since it has the most initial or 'home' investors, who face costs to leave—and

¹²The combination of ex-ante homogeneity and ex-post heterogeneity of investor types is common in the literature on financial market structure (e.g., Duffie et al., 2005; Hugonnier et al., 2019; Maurin, 2022).

 $^{^{13}}$ This is without loss of generality, since regardless of assignment probabilities, any ex-post set of investor types could materialize. For instance, five type a investors and one type b investor could be drawn. Of course, the probability that this happens would be larger if a were more likely to be assigned, but the model is not quantitative in nature.

¹⁴For tractability, we do not incorporate remitted broker profits in investor utility. As investors are not atomistic here, incorporating broker profit leads them to weigh how their actions affect broker profits, which is not a mechanism that pairs well with realistic retail investor choices and significantly complicates the model. Note that in Section 4, the policymaker does weigh both investor utility and broker profits.

Broker 1 has the least. Hence, τ parameterizes both the extent of market power toward investors and its differentiation among brokers. For $\tau \to 0$, Broker 3 has no advantage relative to Broker 1, as investors can freely switch. In contrast, for τ large enough brokers become uncontested monopolists with respect to their initial investors, in which case brokers' relative advantages—bestowed by differences in initial clientele—become unassailable.

2.2.1 Trading modalities

There are up to three types of markets through which brokers can transact to execute trades for their clients. The first type is an intra-broker market. If a given broker's clientele includes some investors of type a and some investors of type b, then the broker can costlessly match pairs of opposing types and execute trades. Up to three such intra-broker markets exist in our model, corresponding to the number of brokers that attract at least two investors. The second type of market is the inter-broker market using the legacy technology. We assume that this is pre-existing, i.e., it does not need to be set up by the brokers and is always present as an option for trades between any pair of brokers. Unlike intra-broker matching, operating on the legacy market is not costless to brokers: a broker pays γ for every investor that it pairs on the legacy market. The third type of market is a tokenized asset market. No such market exists at the beginning of the game and one is only created if two or more brokers choose to form it. Creating a tokenized market comes with total setup cost s to the involved brokers, and we assume that this cost is evenly split among them.¹⁵

The tokenized and legacy markets differ in three ways. First, as described above, the tokenized market costs s to create, while the legacy market has no setup cost. Second, the tokenized market possesses a more cost efficient trading technology. We normalize the variable cost of processing transactions on the tokenized market to zero and thus γ parameterizes the cost efficiency gap between legacy and tokenized markets. Third, the tokenized market has a speed advantage (e.g., reflecting instantaneous settlement): its trading and settlement

 $^{^{15}}$ The coalition formation game is described in Section 2.2.4.

occur before those of the legacy market. 16

We let intra-broker trade occur before the (tokenized and legacy technology) inter-broker markets. The Brokers first clear trades internally and then, for any remaining unmatched clients, seek an opposing type through trade with another broker. In cases where there are multiple possible investor pairings on a given market, we assume that a random draw determines the allocation of trades. For instance, if on the legacy market there is one type a investor and three type b investors, then each type b investor has a one-third chance of being matched with the type a investor.

Which markets are active depends on investor sorting and the formation of the tokenized market. To fix ideas, Figure 2 presents three (among the many) possible configurations. Figure 2a considers an example in which all investors choose the same broker, as shown in Figure 2a. All trading relationships are then intra-broker (shown by thin solid lines), with some investors having incurred extra costs to switch broker (shown by red lines). In Figures 2b and 2c, all investors stick with their initial broker. Intra-broker trade can still occur at either Broker 2 or Broker 3. Inter-broker trade may occur too, either through the legacy market (shown by dotted lines) or through the formation of the tokenized market (shown by thick solid lines). Figure 2b shows the case where both types of inter-broker trade coexist, as only Brokers 1 and 3 form a tokenized market. Figure 2c shows the case where instead all brokers join the same tokenized market, so no trade occurs using the legacy technology.

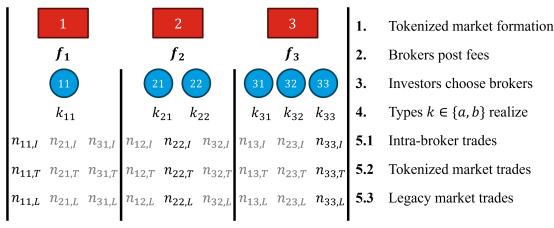
2.2.2 Broker fees

Brokers earn revenues from transaction fees (i.e., bid-ask spreads) f_{lj} on successful matches. We allow brokers full freedom to price discriminate by setting different fees based on an investor's origin l. For example, Broker 1 can set one fee for initial clients, one fee for potential clients switching to it from Broker 2, and one fee for potential clients switching to

 $^{^{16}}$ The timing of the game is shown in Figure 1 and further described in Section 2.3.

¹⁷Broker preference for internal clearing, including in relation to settlement immediacy, has empirical support (Bowman et al., 2024; Lu et al., 2023).

Figure 1: Timing and actions



Notes: This figure summarizes the timing of the game. Red rectangles indicate brokers and blue circles indicate investors. At Stage 5, quantities of trades facilitated by a broker for its initial investors are shown in black, and quantities of trades facilitated for other investors are shown in gray.

Notes: This figure shows some example permutations of (i) investors' choices of broker, and (ii) brokers' choices regarding tokenized market formation. Blue circles represent investors and red rectangles represent brokers. Intra-broker trading relationships are shown by thin solid lines, with red lines denoting new investor-broker pairs, for which the investor incurs a switching cost. Dotted lines show inter-broker relationships in the pre-existing legacy market, and thick solid lines show inter-broker relationships in a tokenized asset market.

it from Broker 3. Each broker j thus posts a menu of fees $\mathbf{f}_j = (f_{1j}; f_{2j}; f_{3j})$ that investors observe before choosing a broker.¹⁸

The mode of competition between brokers can be described as Bertrand competition under incumbency, matching, and marginal cost differentiation. The incumbency differentiation comes from positive τ , which provides a broker with market power in setting fees toward its initial clients. The matching differentiation originates in the heterogeneity in initial broker clientele and in inclusion or exclusion from the tokenized market, both of which can in turn produce differences among brokers in the probabilities, P_{kj} , that investors obtain gains from trade. ¹⁹ Marginal cost differentiation similarly arises from different probabilities of clearing trade on the three markets. Each trade that a broker facilitates on the legacy market costs it γ , whereas trades on the intra-broker and tokenized markets are costless. The expected marginal cost of facilitating trades can thus differ among brokers, which can affect the fees that they offer investors.

2.2.3 Broker profits

Let $n_{lj,p}$ denote the number of trades facilitated by broker j for investors from origin l on market $p \in \{I, T, L\}$, representing the intra-broker, tokenized and legacy markets respectively. Furthermore, $n_{lj} = \sum_{p} n_{lj,p}$ denotes the total number of trades, across all three markets, facilitated by broker j for investors from origin l. We stack these totals from all three origins in the vector $\mathbf{n}_j = (n_{1j}; n_{2j}; n_{3j})$. Lastly, $n_{j,L} = \sum_{l} n_{lj,L}$ denotes the total number of trades facilitated by broker j on the legacy market, across investors from all three origins. The broker's total profit is then:

$$\pi_j = \mathbf{f}_j' \mathbf{n}_j - \gamma \cdot n_{j,L} - s_j \tag{2}$$

¹⁸We specify that fees f_{lj} are always non-negative, reflecting that in well-functioning markets negative bid-ask spreads would be quickly arbitraged away if they arose.

¹⁹A broker that services more investors has a greater chance of matching its investors intra-broker. And a broker that is part of the tokenized market has an additional probability of matching investors, compared to a broker whose only access to other brokers is through the legacy market.

where

$$s_j = \mu_j \frac{s}{m} \tag{3}$$

is the potential cost to broker j of forming a tokenized market, where m is the number of brokers participating and μ_j is a dummy taking value one if broker j is involved in forming such a market and zero otherwise.

2.2.4 Tokenized market formation

The game begins with an opportunity for a coalition of brokers to form a tokenized market. We allow for an arbitrarily long period of negotiation, and simply require that, for a coalition structure that emerges to be an equilibrium, no broker or coalition of brokers would strictly prefer to deviate from their existing coalition.

Formally, denote by $J = \{1, 2, 3\}$ the set of brokers in the market and denote by \mathcal{P} the set of all possible coalition structures, i.e., all partitions of J.²⁰ Each coalition structure $\mathcal{C} \in \mathcal{P}$ is associated with a single vector $\boldsymbol{\pi}^e(\mathcal{C}, s, \gamma) \in \mathbb{R}^3$ of expected payoffs $\pi_j^e(\mathcal{C}, s, \gamma)$ for each broker j, given observed cost parameters s and γ , where expectations are formed over the distribution of investor types at a subsequent stage. A coalition structure \mathcal{C} is then an equilibrium if there does not exist any alternative coalition structure \mathcal{C}' containing a new coalition $\mathcal{C}' \notin \mathcal{C}$ such that $\pi_j^e(\mathcal{C}', s, \gamma) > \pi_j^e(\mathcal{C}, s, \gamma) \ \forall \ j \in \mathcal{C}'$. A tokenized market is formed when an equilibrium contains a coalition of more than one broker. For convenience, we attach the terms 'No Coalition' and 'Grand Coalition' to, respectively, the coalition structures $\{\{1\}, \{2\}, \{3\}\}$ and $\{1, 2, 3\}$.

 $^{^{20}}$ Intuitively, each coalition structure is a set of non-empty coalitions such that each broker is included in exactly one coalition. For our three-broker case, there are five such coalition structures, so we have: $\mathcal{P} = \{\{\{1\}, \{2\}, \{3\}\}, \{\{1,2\}, \{3\}\}, \{\{1,3\}, \{2\}\}, \{\{1\}, \{2,3\}\}\}\}$. The first of these is the case where no tokenized market forms, the second to fourth structures represent the partial coalitions of two brokers, and the last structure is the three-broker coalition.

²¹For instance, the structure containing the three-broker coalition ($\mathcal{C} = \{\{1,2,3\}\}$) is an equilibrium if: (i) no individual broker would strictly prefer to use only the legacy market for inter-broker trade, and (ii) no pair of brokers would both strictly prefer to form their own tokenized market that excludes the third broker.

2.3 Timing

The timing of agents' actions and the realization of events is shown in Figure 1. The game takes place over the following five stages. In Stage 1, a coalition of brokers (possibly) forms a tokenized market. In Stage 2, brokers post their menus of fees to investors. In Stage 3, investors choose their brokers. In Stage 4, investors' types realize. Finally, in Stage 5, brokers execute trades—first intra-broker, then on the tokenized market (if it exists), and lastly on the legacy market.

Only the first three stages of this game are decision stages. In the first two stages, brokers choose coalitions in the tokenized market formation game and then set their fees f_j . In the third stage, investors choose which broker to sign up with. The fourth and fifth stages are realization stages. When types are realized in Stage 4, this reveals potential trades. The set of trades that actually occurs then results from brokers' sequential execution of trades in Stage 5—first clearing intra-broker matches, then matches on the tokenized market, and finally on the legacy market.

2.4 Optimization problems

To prepare for solving the game by backward induction, we present the agent optimization problems in the reverse order of the decision stages. At Stage 3, each investor li chooses its broker j_{li} to maximize the expected value of its payoff u_{li} , taking into account the (observed) coalition structure \mathcal{C} and the (observed) fees of all brokers $\mathbf{f} = (\mathbf{f}_1; \mathbf{f}_2; \mathbf{f}_3)$:

$$\max_{j_{li}} \{ u_{li}^e(\mathcal{C}, \boldsymbol{f}) \} . \tag{4}$$

At Stage 2, each broker j sets its menu of fees f_j to maximize its expected profits:

$$\max_{\boldsymbol{f}_i} \{ \pi_j^e(\mathcal{C}) = \boldsymbol{f}_j' \mathbf{n}_j^e(\mathcal{C}, \boldsymbol{f}) - \gamma \cdot n_{j,L}^e(\mathcal{C}, \boldsymbol{f}) - s_j \}$$
 (5)

Here, the broker takes into account the observed coalition structure C from Stage 1 and foresees how its fees will subsequently affect investors' choices of broker at Stage 3, and hence the broker's expected number of trades at Stage 5.²²

In Stage 1 brokers backward induce all subsequent stages of the game and bargain over coalition structures \mathcal{C} in an attempt to maximize their expected profits $\pi_j^e(\mathcal{C})$.

2.5 Constraints

We aim to analyze the incentives of competitive brokers, each with a degree of market power, to jointly form a tokenized market. With this in mind, we center attention on cases that do not produce broker monopolies. Such monopolies arise at two opposite extremes of our modeling environment: when switching costs are so large that investors are walled in at their initial broker, and when such costs are so small that investors agglomerate at a single broker.

2.5.1 Three uncontested monopolies

If switching costs are so large that brokers become uncontested monopolists toward their investor bases, brokers will set fees equal to investors' gains from trade, regardless of the coalition structure. Fees therefore no longer endogenously respond to coalition structures, depriving the model of an important margin of broker optimization and adjustment to coalitions. The constraint $\tau \leq \frac{2}{3}$ suffices to exclude uncontested monopolies.

2.5.2 Single monopolist

We aim to spotlight cases where the market for brokerage is somewhat contestable, but not so contestable that brokers' incentives shift away from coalition formation toward intra-broker unification. This is essentially another form of monopolization, this time by attracting all investors to the same broker (as in Figure 2a, for example). We therefore also impose that

The expectations operator on \mathbf{n}_{j}^{e} and $n_{j,L}^{e}$ reflects that at Stage 2 investor types—which will be drawn at Stage 4—affect matching at Stage 5.

 $\tau \geq \frac{1}{3}$. This lower bound on τ ensures that the equilibrium wherein all investors stay at their initial brokers is always feasible, where feasibility here means that investors' priors are self-confirming (i.e., given that each investor believes that every other investor will stay at their initial broker, each investor optimally chooses to stay at its initial broker).

Investors' selection of brokers (Stage 3) lends itself to multiple equilibria because investors impose externalities on each other. The more investors pool on one broker, the more attractive that broker is to other investors, given that brokers give priority to (costless) intra-broker trades. For example, for a given parameterization the following equilibria could co-exist: first, an equilibrium where each investor acts from the prior that all other investors will stay at their initial brokers and this prior is confirmed by each investors' decision to stay, as in Figures 2b and 2c; second, an equilibrium where each investor expects all other investors to move to (or stay at) Broker 3 and, given this prior, all investors indeed aggregate at Broker 3, as in Figure 2a.

We assume that whenever the equilibrium wherein all investors stay at their initial brokers is feasible, it comes about. That is, for parameterizations that give rise to multiple equilibria at Stage 3, we utilize this as an equilibrium selection criterion. In addition to matching the empirical observation of non-monopolization on brokerage markets, this criterion is consistent with the rise of passive investing—where investors' choice of broker can be sticky (e.g., maintaining the same index-tracking fund for many years)—as well as the far larger number of retail investors than brokers, which makes coordinating all investors to pool on one broker difficult. More generally, the notion that agents are biased toward expecting the persistence of the initial distribution—i.e., status quo bias—is well established (Battigalli et al., 2015; Fudenberg and Levine, 2016; Guney and Richter, 2018).

2.5.3 Other constraints

To facilitate the derivation of analytical solutions for optimal broker fees, we constrain the cost of clearing legacy market transactions to be no more than one-sixth of an investor's gains

from trade: $0 < \gamma \le \frac{1}{6}$.²³ Furthermore, to ensure that investors have enough endowment to cover fees (and also to always retain positive utility if they incur non-pecuniary cost τ), we set $\eta \ge 2$. Finally, to avoid discontinuities on indifference thresholds, we introduce tie-breaking assumptions in favor of (larger) tokenized market formation when brokers are exactly indifferent between coalitions.²⁴

3 Equilibrium

This section presents the equilibrium of the game described in Section 2, which Lemma 1 shows depends on the tokenized market setup cost s and the legacy market trading cost γ .²⁵

Lemma 1 (Baseline equilibrium) If $s > \frac{1}{8} + \frac{7}{8}\gamma$, no tokenized market forms in equilibrium. If $s \leq \frac{1}{8} + \frac{7}{8}\gamma$, Brokers 1 and 3 form a tokenized market.

Proof of Lemma 1. See Appendix A.1 (p. i). ■

Figure 3a depicts these regions. Above the blue line, no multi-broker coalition forms (i.e., no tokenized market is created) because the setup cost is too high relative to the potential benefits. On and below the blue line, the outcome reflects two forces. First, tokenized markets in general are more attractive, because the setup cost is lower relative to efficiency gains vis-à-vis the legacy market (i.e., γ). Second, forming a partial coalition with two brokers diverts trade from the excluded broker toward the participating brokers. To see this, note that the 1 & 3 Coalition forms even as γ approaches zero, as long as s is not too high.

²³While poaching investors from another broker allows the gaining broker to utilize intra-broker clearance on more trades, it can also lead to more legacy market transactions for this broker, depending on how many investors remain at other brokers. When legacy market transactions become too costly, this aspect complicates the determination of fee equilibria. We note that setting $\gamma \leq \frac{1}{6}$ incurs no qualitative loss of generality, as seen in Section 4 where all relevant policy zones are represented.

²⁴When brokers are indifferent between a coalition of three and another option (either a smaller coalition or No Coalition), then they choose the coalition of three. When two brokers are indifferent between forming a coalition of two and No Coalition then they form the coalition of two. These tie-breaking assumptions are without loss of generality—none of our results would be overturned by reversing them.

²⁵The other two exogenous parameters—the endowment η and the switching cost τ —do not influence equilibrium outcomes, subject to the constraints discussed in Section 2.5.

Here, forming a tokenized market offers no reduction in costs—since all trades are already costless—but it still provides a benefit to participating brokers in the form of preferential access to the other's investors, due to the faster execution of tokenized market trades in Stage 5. This raises the matching probabilities offered by the participating brokers (at the expense of the excluded broker), in turn allowing them to charge higher fees.

Among the three feasible partial coalitions (1 & 2, 1 & 3, 2 & 3), why does the 1 & 3 Coalition form? Brokers 2 and 3 are each other's fiercest competitors: their larger initial clienteles than Broker 1 mean that, ceteris paribus, they offer better matching odds than Broker 1, due to intra-broker clearing. Thus, in the competition for Broker 2's initial clients, Broker 3 binds how high Broker 2 can set its fees given switching costs and, similarly, Broker 2 binds Broker 3's fees. Forming a partial coalition with Broker 1, gives each the opportunity to weaken the other's position, since an excluded broker offers a worse matching probability to investors. Hence, when either Broker 2 or 3 joins a partial coalition with Broker 1, it expects to process more trades and at higher equilibrium fees. Broker 1 also benefits from trade diversion and this benefit is largest when forming a partial coalition with Broker 3, because this gives it the best chance to match its investor on the tokenized market.

4 Welfare

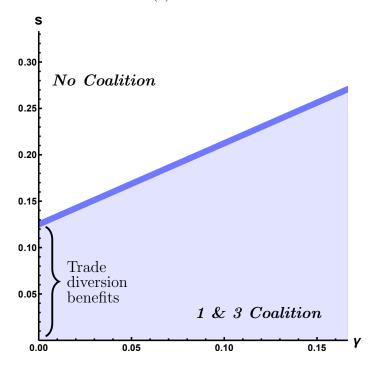
Denote the total expected profit of the brokerage sector by $\Pi^e = \sum_{j=1}^3 \pi_j^e$, and the total expected utility of the investors by $U^e = \sum_{li} u_{li}^e$. Total expected welfare is then given by $W^e = \Pi^e + U^e$. For convenience, we let 'welfare' to refer to total *expected* welfare.

Lemma 2 (Optimal outcomes) If $s > \frac{13}{8}\gamma$, welfare is maximized when no tokenized market exists. If $s < \frac{13}{8}\gamma$, welfare is maximized by a tokenized market that includes all brokers. If $s = \frac{13}{8}\gamma$, welfare is equal under both outcomes.

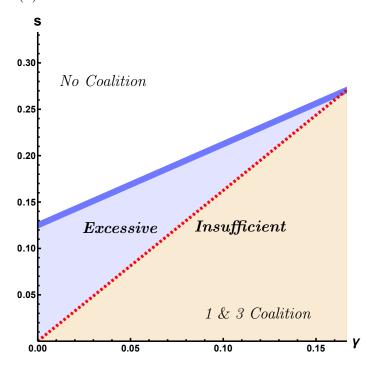
Proof of Lemma 2. See Appendix A.2 (p. ix).

Figure 3: Equilibrium without policy

(a) Baseline



(b) Excessive investment and insufficient tokenization



Notes: These figures depict the baseline equilibrium as a function of the tokenized market setup cost s and the legacy market trading cost γ . In Panel (a), above (below) the thick blue line the equilibrium features No Coalition (the 1 & 3 Coalition). In Panel (b), below the thick blue line and to the left (right) of the dotted red line, this reflects excessive (insufficient) coalition formation.

Proposition 1 (Excessive investment and insufficient tokenization) The private equilibrium can feature excessive investment and insufficient tokenization. Investment is excessive if $\frac{13}{8}\gamma \leq s \leq \frac{1}{8} + \frac{7}{8}\gamma$. Tokenization is insufficient if $s \leq \frac{13}{8}\gamma$.

Proof of Proposition 1. The result follows immediately from comparing the regions defined in Lemmas 1 and 2. ■

Figure 3b illustrates Proposition 1. Below the red dashed line, identified by Lemma 2, the Grand Coalition is socially optimal because the efficiency benefits of tokenization outweigh the costs and the gap between these benefits and costs is largest in the Grand Coalition where costly legacy market trade is fully eliminated. Above this line, setup costs are too high and No Coalition is socially optimal.²⁶ We also note that investor utility makes the red dashed line run higher than if only broker profits entered welfare, because part of the efficiency gains of tokenization endogenously pass on to investors in the form of lower equilibrium fees.

No Coalition is socially optimal in Figure 3b's white and blue regions, but in the latter brokers form the 1 & 3 Coalition. Trade diversion benefits create private incentives that result in excessive investment: s is incurred to set up a tokenized market, but welfare would be higher if instead brokers relied only on the legacy market. In contrast, in the yellow region it is socially optimal to incur s and set up a tokenized market, but the coalition that forms privately is too narrow: Brokers 1 & 3 divert some trades from Broker 2 but remaining trades with that broker occur at the (socially wasteful) higher legacy market cost γ .

 $^{^{26}}$ Note that the red dashed line passes through the origin: without efficiency gains any setup cost is too high, and without a setup cost any efficiency gains justify creating the Grand Coalition. We further note that the parameter η also affects welfare (which trivially increases for a higher η) but does not affect broker behavior. A higher τ does not affect aggregate welfare because in equilibrium all brokers retain their initial investors and τ is not incurred. Higher τ does however affect the distribution of welfare between investors and brokers, since a higher switching cost allows brokers to extract higher fees from their initial investors.

5 Policy analysis

In the context of our model, a policymaker can straightforwardly attain the social optimum by forbidding tokenization above the red line in Figure 3b and mandating the Grand Coalition below it. But political economy often precludes such heavy-handed approaches. This section considers whether a policymaker can maximize social welfare by less intrusive means. In particular, we center attention on the two types of policies seen in the initiatives discussed on p.1: an interoperability mandate and a public-private partnership to form a tokenized market. Among these two, the interoperability mandate is arguably the lighter-touch policy because it merely requires that brokers keep the door open to each other when creating a new ledger, while a public-private partnership involves fiscal resources to help create the tokenized market. We therefore start our discussion from the interoperability mandate and then ask whether and when the model indicates that it should be supplemented by (the minimum necessary) public cost-sharing.

We incorporate the interoperability mandate—introduced prior to Stage 1—as a requirement that any coalition of two brokers deciding to form a tokenized market must offer the third broker the opportunity to join at no cost. We break Stage 1 into three sub-stages. At Stage 1.1, each broker j sets a budget limit of s_j^{max} that it is willing to invest in tokenization projects. At Stage 1.2, coalitions of brokers can form tokenized markets by each investing $s_j \leq s_j^{max}$, as described in Section 2.2. Finally, if a coalition C of two brokers formed at Stage 1.2, Stage 1.3 occurs, in which the excluded broker $k \notin C$ is offered the opportunity to join C at no cost (i.e., $s_k = 0$).²⁷ With this setup we derive the following results:

 $^{^{27}}$ We include Stage 1.1 to avoid a free-rider problem that can otherwise emerge in Stage 1.2. In the absence of upper limits on s_j , each broker has an incentive to block the Grand Coalition in the hope of forcing the other brokers to bear the full cost of setting up a tokenized market that the excluded broker will ultimately be able to join for free in Stage 1.3. Since all brokers reason the same, this strategy is never successful, but it can also preclude any equilibrium from forming. Stage 1.1 arguably speaks to real-world possibilities to commit, since brokers could credibly convey a limit on how much they are willing to invest in tokenization (i.e., $s_j^{max} \leq s/3$ in our case). Note that this Stage 1.1 is without loss of generality versus the baseline, since a model featuring Stages 1.1 and 1.2 alone (without the interoperability mandate introduced at Stage 1.3) results in identical coalition structures to the simpler combined Stage 1 described in Section 2.3.

Lemma 3 (Interoperability mandate and excessive investment) An interoperability mandate prevents the excessive investment that occurs in equilibrium when $\frac{13}{8}\gamma \leq s \leq \frac{1}{8} + \frac{7}{8}\gamma$.

Proof of Lemma 3. See Appendix A.3 (p. x).

The interoperability mandate counteracts trade diversion: partial coalitions are no longer sustained in equilibrium because the participating brokers pay $\frac{s}{2}$ to form a coalition that the excluded broker subsequently decides to join for free. Brokers thus come to choose between No Coalition or the Grand Coalition.²⁸ In the blue region in Figure 4a at least one broker is never willing to form the Grand Coalition, so No Coalition results. The blue zone is where investment is excessive in the private equilibrium (as in Figure 3b) and therefore the interoperability mandate here leads to the socially optimal outcome of No Coalition.

Lemma 4 (Interoperability mandate and insufficient tokenization #1) An interoperability mandate prevents the insufficient tokenization that results in equilibrium when $s \leq \frac{27}{20}\gamma$.

Proof of Lemma 4. See Appendix A.4 (p. xiii). ■

This result relates to the green region in Figure 4a, where tokenization is insufficient in the private equilibrium (Figure 3b). The green line shows the boundary below which all brokers prefer the Grand Coalition to No Coalition.

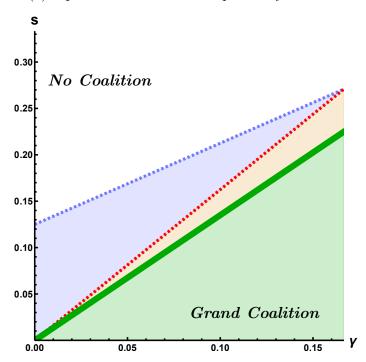
Lemma 5 (Interoperability mandate and insufficient tokenization #2) An interoperability mandate does not prevent the insufficient tokenization that results in equilibrium when $\frac{27}{20}\gamma < s \leq \frac{13}{8}\gamma$.

Proof of Lemma 5. See Appendix A.5 (p. xiii). ■

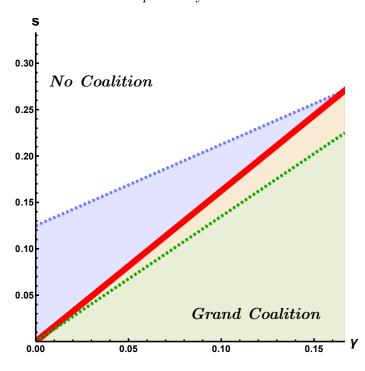
²⁸In this section, we use 'Grand Coalition' to refer only to the 'Egalitarian Grand Coalition' that is formed when all brokers contribute $\frac{s}{3}$ —i.e., this label excludes a tokenized market encompassing three brokers but paid for by only two.

Figure 4: Equilibria with policy

(a) Equilibrium with an interoperability mandate



(b) Equilibrium with an interoperability mandate and tokenization subsidy



Notes: Panel (a) depicts the equilibrium of the model in the presence of an interoperability mandate. Above the thick green line this features No Coalition, while below the thick green line it features the Grand Coalition. Panel (b) depicts the equilibrium in the presence of both an interoperability mandate and (in the yellow region) a tokenization subsidy. Above the thick red line this equilibrium features No Coalition, while below the thick red line it features the Grand Coalition.

This result, which relates to the yellow region in Figure 4a, has the same intuition as Lemma 3, but occurs in a region where the private equilibrium produces insufficient tokenization rather than excessive investment in Figure 3b. Like in Lemma 3, at least one broker vetoes the Grand Coalition so that No Coalition ensues.²⁹

Lemma 5 highlights a deeper tension: the social planner favors the Grand Coalition more than does the broker who is least keen to join it. The social planner weighs aggregate broker profits, as well as investor utility. Instead, the private equilibrium only produces the Grand Coalition when the *least amenable* broker prefers it to No Coalition. The resulting 'underinvestment problem' can only be solved by policy that changes the least amenable broker's valuation of the Grand Coalition outcome.

We therefore next turn to a combination of the interoperability mandate with public cost sharing, which we introduce in the form of a tokenization subsidy. This subsidy $\sigma > 0$ is announced before Stage 1 and reduces the setup cost from s to $s - \sigma$. The subsidy is funded with a lump-sum tax on investor endowments. By itself (without interoperability) such a subsidy is not a useful policy in the context of our model. This can be seen from Figure 3b, where the outcome at each point (γ, s) in the chart is replaced by $(\gamma, s - \sigma)$ vertically below. Below the blue line, this never changes the coalition structure—regardless of the size of σ —since the 1 & 3 Coalition always forms. Above the blue line, the equilibrium is already socially optimal and there is nothing for the subsidy to improve upon. Thus, no simple tokenization subsidy can achieve the social optimum when used alone.³⁰

In our model, combining an interoperability mandate with a tokenization subsidy instead

²⁹Interestingly, in the yellow region the veto is always cast by Broker 2—the very broker that is excluded from the tokenized market without the interoperability mandate. Broker 2 prefers the Grand Coalition to the 1 & 3 Coalition. But after interoperability removes the latter as a feasible equilibrium, Broker 2 is the least inclined toward the Grand Coalition in comparison to No Coalition. Broker 3 can spread the setup cost of the Grand Coalition over the largest number of expected transactions. Per expected transaction, however, Broker 1 sees the largest cost reductions from joining the Grand Coalition because without tokenization it processes all transactions on the legacy market. For Broker 2, these expected cost reductions per transaction are smaller, while it also has fewer expected transactions over which to spread the setup cost than Broker 3.

 $^{^{30}}$ A sufficiently large tax ($\sigma < 0$, representing, e.g., stringent regulatory or licensing requirements) could prevent excessive investment (blue zone of Figure 3b) by offsetting the gains from trade diversion that otherwise motivate Brokers 1 and 3 to form a tokenized market. However, such a tax cannot resolve insufficient tokenization, as there is no point in Figure 3b—across any s—at which brokers form the Grand Coalition.

allows the policymaker to sufficiently incentivize the marginal broker to join the Grand Coalition in the yellow region of Figure 4a. We derive the following result:

Lemma 6 (Interoperability mandate plus tokenization subsidy) An interoperability mandate combined with a tokenization subsidy $\sigma \geq \frac{11}{40}\gamma$ prevents the insufficient tokenization that results in equilibrium when $\frac{27}{20}\gamma < s \leq \frac{13}{8}\gamma$.

Proof of Lemma 6. See Appendix A.6 (p. xiv). ■

In Figure 4a, all points in the yellow region are directly above a point in the green region where the interoperability mandate produces the Grand Coalition. Hence, in all such cases a sufficiently large subsidy produces the Grand Coalition by moving the equilibrium point from (γ, s) to $(\gamma, s - \sigma)$. In combination with Lemmas 3 and 4 above, we thus have:

Proposition 2 (Optimal policy) A policymaker with the ability to impose an interoperability mandate and subsidize the creation of tokenized markets can always achieve the socially optimal outcome. Specifically, it can achieve this by:

- 1. Taking no action (laissez-faire) if $s > \frac{1}{8} + \frac{7}{8}\gamma$,
- 2. Imposing an interoperability mandate if $s \leq \frac{1}{8} + \frac{7}{8}\gamma$, and
- 3. Subsidizing the formation of tokenized markets if $\frac{27}{20}\gamma < s \leq \frac{13}{8}\gamma$, where the minimum needed size of the subsidy is $\sigma^{min} = \frac{11}{40}\gamma$.

Proof of Proposition 2. When $s > \frac{1}{8} + \frac{7}{8}\gamma$, No Coalition maximizes welfare (Proposition 1) and laissez-faire is optimal. Policies 2 and 3 result from combining Lemmas 3, 4, and 6.

Equilibrium outcomes under this policy package are shown in Figure 4b. In the white region, No Coalition is both the private equilibrium and optimal, so the policymaker takes no action. In the blue region, the private equilibrium leads to excessive investment, but the interoperability mandate produces No Coalition, the optimal outcome. Below the red

line, the Grand Coalition is optimal, and this is achieved by the interoperability mandate alone (in the green region) or in combination with a sufficiently large subsidy (in the yellow region).

6 Extensions

In this section, we consider two variations on the baseline model. First, we generalize brokers' possible actions during the coalition formation game by allowing them to make side-payments to one another. Second, we allow for the existence of a pre-existing, independent tokenized market, such as on a public blockchain.

6.1 Allowing side-payments

In the baseline model, brokers only bargain over coalition structures. This section generalizes their negotiation space by allowing them to simultaneously make side-payments to one another.³¹ For instance, Broker 2 can now pay Brokers 1 and 3 to form the Grand Coalition instead of excluding it by forming the 1 & 3 Coalition. Alternatively, Broker 2 could pay Brokers 1 and 3 not to form any coalition. We define an outcome as a coalition structure and an associated trio of net side-payments between brokers. Such an outcome is an equilibrium if no broker or coalition of brokers would strictly prefer to deviate from the existing coalition structure to an alternative coalition structure that is supported by feasible side-payments.

Formally, we modify the tokenized market formation game described in Section 2.2.4 as follows. Define vector $\mathbf{v} = (v_{12}; v_{13}; v_{23})$ as a series of net side-payments v_{jk} from each broker j to each other broker k.³² An outcome $(\mathcal{C}, \mathbf{v})$ is then the pair of a coalition structure \mathcal{C} with a trio of net side-payments \mathbf{v} . Each such pair is associated with a single vector of expected

³¹This also effectively relaxes the assumption of an equal distribution of setup costs among coalition members, since side-payments can deliver any distribution across a coalition's members of the net gains from tokenization.

³²For notational convenience below, we also denote the same net transfers measured in the opposite direction by $v_{kj} \equiv -v_{jk}$.

broker payoffs $\boldsymbol{\pi}^{e}(\mathcal{C}, \mathbf{v}, s, \gamma) \in \mathbb{R}^{3}$, given observed cost parameters s and γ . Such an outcome $(\mathcal{C}, \mathbf{v})$ is an equilibrium if there does not exist any alternative pair $(\mathcal{C}', \mathbf{v}') \neq (\mathcal{C}, \mathbf{v})$ where: (i) \mathcal{C}' contains a new coalition $\mathcal{C}' \notin \mathcal{C}$ such that $\pi_{j}^{e}(\mathcal{C}', \mathbf{v}', s, \gamma) > \pi_{j}^{e}(\mathcal{C}, \mathbf{v}, s, \gamma) \ \forall j \in \mathcal{C}'$, and (ii) $\pi_{j}^{e}(\mathcal{C}', \mathbf{v}', s, \gamma) > \pi_{j}^{e}(\mathcal{C}, \mathbf{v}, s, \gamma)$ for all j for whom $\sum_{k} v'_{jk} > 0$.³³ Using this definition we derive the following result:

Proposition 3 (Welfare with side-payments) When brokers can make side-payments, insufficient tokenization can occur so the private equilibrium does not always maximize welfare.

Proof of Proposition 3. See Appendix A.7 (p. xiv). ■

Intuitively, the ability to make side-payments allows brokers to act collectively to maximize their joint surplus. Excessive investment never occurs, since any trade diversion incentives can be outweighed by side-payments from the broker that would otherwise lose out. However, brokers' joint profit maximization does not align perfectly with social welfare. Under the Grand Coalition, competition between brokers rises relative to the baseline (i.e., relative to the situation without side-payments, when the 1 & 3 Coalition forms), so average broker fees are lower, benefiting investors at the expense of brokers. Thus part of the gains from the Grand Coalition flow to investors, implying that even with fully flexible side-payments an underinvestment problem can persist. Thus for some parameter values our model indicates that a policy response remains necessary to achieve the social optimum:

Proposition 4 (Policy implications of side-payments) With fully flexible side-payments, a sufficiently large tokenization subsidy can always maximize welfare.

 $^{^{33}}$ Intuitively, equilibrium requires that no alternative outcome exists in which: (i) a broker or coalition of brokers C' chooses to deviate from their existing coalition, and (ii) any broker—including one excluded from C'—that makes net payments to the other brokers is willing to do so. Note also that this definition allows a range of equilibrium outcomes to exist for each coalition structure: a given C could be supported by many different vectors of side-payments \mathbf{v} , reflecting different ways of dividing the surplus created, conditional on no broker or coalition of brokers receiving so little or paying so much that they would prefer to deviate.

6.2 Independent public ledger infrastructure

The baseline model assumes that brokers invest s to establish a tokenized market on a proprietary ledger. An alternative is to use a pre-existing public blockchain, such as the Ethereum ledger, which can support trade in tokenized assets. We therefore extend the model to include the existence of an independent ledger infrastructure (ILI) that any broker can freely join and that offers an inter-broker trading cost, $\gamma' \in [0, \gamma)$.³⁴ We assume that each broker decides whether to join the platform at the beginning of the game, before Stage 1, and that trades on the ILI clear after the proprietary tokenized market but before the legacy market.³⁵ We derive the following results:

Proposition 5 (Independent ledger infrastructure) The availability of a sufficiently efficient independent ledger infrastructure leads the private equilibrium to produce the socially optimal outcome if and only if tokenized market setup costs are high. Specifically:

- 1. When $s > \frac{1}{8}$, there always exists an (ILI versus legacy market) efficiency gain $\gamma \gamma'$ that leads the private equilibrium to produce the socially optimal outcome.
- 2. When $s \leq \frac{1}{8}$, there is no efficiency gain $\gamma \gamma'$ that leads the private equilibrium to produce the socially optimal outcome.

Proof of Proposition 5. See Appendix A.9 (p. xv). ■

Since the ILI comes with lower transaction costs than the legacy market and is costless to join, all brokers do so. The legacy market is therefore displaced: brokers clear on the ILI any

³⁴We thus allow for the possibility that the ILI clears trades at positive cost because, relative to brokers' proprietary ledger, transaction validation on a public blockchain can involve substantial costs (e.g., Ethereum gas fees).

³⁵This reflects that the new infrastructure is faster than the legacy market infrastructure, but also that brokers choosing to set up their own tokenized market could (and would) still choose to favor those trades on their private tokenized market over those on the public ILI.

trades remaining after the intra-broker and proprietary tokenized markets have settled. This necessarily raises aggregate welfare by reducing trading costs. However, whether the shift achieves the *optimal* outcome depends on the initial level of s. When forming a proprietary tokenized market is sufficiently costly relative to the potential gains from trade diversion, Broker 1 and Broker 3 are amenable to abandoning their private coalition and conducting all their inter-broker trades on the ILI. In contrast, when s is sufficiently low even an ILI with a trading cost of $\gamma' \to 0$ does not induce Brokers 1 and 3 to forego the gains from trade diversion that come with forming their own exclusive market. When the presence of the ILI fails to move the private equilibrium to the social optimum, optimal policy remains as in Proposition 2:

Proposition 6 (Policy implications of independent ledger infrastructure) When $s \le \frac{1}{8} + \frac{7}{8}\gamma'$, the optimal combination of an interoperability mandate and public-private cost-sharing achieves the social optimum. This is not always attainable when either policy instrument is used in isolation.

Proof of Proposition 6. See Appendix A.10 (p. xv). ■

7 Conclusion

As more brokers explore the tokenization of financial assets, policymakers seek to reap the potential benefits that this technology offers while mitigating the risk of market fragmentation. This paper examines the drivers of such fragmentation and provides the first formal framework for analyzing optimal policy responses.

We model the endogenous joint adoption of a new, more efficient trading technology by brokers already connected through a legacy trading platform. Brokers with heterogeneous market power compete to attract investors and execute their trades. Coalitions of brokers can invest in creating a tokenized market that enables faster and cheaper settlement between them. Trade diversion incentives result in equilibrium coalition structures that feature excessive investment or insufficient tokenization.

Turning to interventions currently being explored by policy institutions, as considered through the lens of our model, we find that neither an interoperability mandate nor public cost-sharing always achieve the socially optimal outcome when implemented in isolation. An interoperability mandate reduces the private return to forming a tokenized market, since the inability to exclude other brokers precludes trade diversion. This disincentive addresses excessive investment. However, when insufficient tokenization is the problem, the disincentive from interoperability can be too strong and instead cause an underinvestment problem. In contrast, public cost-sharing can stimulate investment by reducing the burden falling on brokers, but does not affect the trade diversion incentives that produce exclusive coalitions. Our model indicates that, in combination, the two policies give the policymaker both a 'carrot' and a 'stick' to achieve the optimal degree of tokenization in all cases.

Public cost-sharing also speaks to indirect subsidization through the provision of a central bank-issued means of payment for tokenized financial markets. Tokenized money is needed to enable trading on a market with tokenized assets (Chiu and Monnet, 2024). Several policy initiatives (e.g., Australia, Brazil, Switzerland) currently contemplate the provision of a Central Bank Digital Currency (CBDC) as a means to settle transactions on a tokenized market, which could reduce the barriers for the private sector to tokenize (BIS, 2025; Maechler and Wehrli, 2021).

Future work could build on our framework to incorporate microfoundations for means of payment choice on a tokenized market or for the benefits of tokenized assets' programmability that we have modeled in simplified form. Future work could also consider alternative broker strategies, such as selling access to a tokenized market if a broker can gain a first-mover advantage in tokenization, or the monetization of transaction data generated on such a market. The interaction between brokers' incentives to tokenize and information asymmetries

³⁶Akin to the monetization of transaction data by monopolistic payment platforms (Agur et al., 2025a).

(either among brokers or between investors and brokers) could also be explored.

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Appendix

A Proofs

This appendix contains proofs omitted from the main text. Full calculations for all steps are performed in a *Mathematica* file that is available on request. That file performs all calculations in exact form, while here we round quantities to two decimal places for conciseness and show the exact (fraction) form only for key results.

A.1 Proof of Lemma 1

A.1.1 Solution approach.

We solve for the equilibrium of the model by backward induction through the decision stages, while starting from the prior that all investors choose to stay at their initial broker. If, based on such a status-quo prior, a unique fulfilled-expectations Subgame-Perfect Nash Equilibrium (SPNE) is found then, regardless of whether other equilibria can exist, the selection criterion in Section 2.5.2 implies that this "Stay Equilibrium" is selected. Thus, our solution technique uses the following approach:

- 1. At Stage 3, under the status-quo prior about other investors' behavior, each investor chooses a broker based on observed broker fees and expected match probabilities.
- 2. At Stage 2, each broker determines its fees, internalizing how these affect investor sorting at Stage 3. At this stage, brokers take the coalition structure as given.
- 3. At Stage 1, brokers negotiate about coalition formation, foreseeing the impact of each coalition structure on fees and investor sorting. We compare the expected payoffs $\pi_j^e(\mathcal{C}, s, \gamma)$ of each broker across coalition structures to identify the coalition structure that results in Stage 1, for each combination of s and γ .
- 4. If this backward induction results in a unique SPNE that, moreover, is consistent with the status-quo prior (i.e., in which at Stage 3 each investor indeed opts to stay at its initial broker), then our derivations end.

Since we will find below that a fulfilled-expectations SPNE based on the status-quo prior always exists, no additional steps (identifying how to proceed in the absence of such a SPNE) to the above algorithm need to be specified.

A.1.2 Detailed proof.

Stage 3. In the final decision stage of the game, each investor li chooses a broker j based on brokers' fees f and the investor's expected match probabilities P_{aj}^e and P_{bj}^e in the event that they join j and are assigned each type:

$$u_{li}^{e}(\mathcal{C}, \mathbf{f}) = \eta + \max\{(1 - f_{lj}) \left(\frac{1}{2} P_{aj}^{e} + \frac{1}{2} P_{bj}^{e}\right), 0\} - \tau_{lj}$$
 (A.1.1)

Since each investor is equally likely to be assigned type a as type b, the expected overall distribution of types (e.g., six a, five a and one b, four a and two b, ...) is symmetric in a and b, so we can write $P_j^e := P_{aj}^e = P_{bj}^e$. The expected match probability for an investor choosing broker j depends on two things: the (observed) coalition structure \mathcal{C} and the distribution of investors across brokers. We denote the latter by Φ , defined as a vector whose six elements correspond to the brokers chosen by each investor li, ordered by l and then i. For example, if $\Phi' = (1, 2, 2, 3, 3, 3)$ then we have that each investor stays with their initial broker. For convenience below, we denote this particular example the "Stay Distribution", Φ^{Stay} . Since investors choose brokers simultaneously, each investor makes their decisions only on the basis of their expectations of other investors' broker choices. Thus we further decompose Φ into two elements: the broker choice $j_{li} \in \{1,2,3\}$ of the investor li whose choice we are considering, and that investor's expectations Φ_{li}^e of the choices of all the other investors. As discussed in our solution technique, we initially confine our attention to cases in which each investor expects all other investors to stay with their initial broker, which we denote by $\Phi_{li}^e = \Phi_{li}^{Stay}$. For example, $\Phi_{11}^{Stay} = (j_{11}, 2, 2, 3, 3, 3)$, where j_{11} is the broker choice of investor 11, while that investor acts from the prior that the other investors stay at their initial brokers. Moreover, since investors within a given broker l are identical before (Stage 4) type assignment, the expected payoff for each is identical, allowing us to drop the subscript i. Equation A.1.1 becomes:

$$u_l^e(\mathcal{C}, \mathbf{f}) = \eta + \max\{(1 - f_{lj})P_j^e(\mathcal{C}, j_l, \mathbf{\Phi}_l^{Stay}), 0\} - \tau_{lj}.$$
 (A.1.2)

We first derive the match probabilities $P_j^e(\mathcal{C}, j_l, \mathbf{\Phi}_l^{Stay})$. These can be calculated mechanically in three steps. First, we calculate the relative probability of different numbers of investors being assigned to type a versus b at Stage 4. Since each investor can draw one of two types and investors' draws are independent, there are $2^6 = 64$ possible outcomes. The distribution of the number of type a investors follows from a straightforward application of the binomial distribution and is shown in Table A.1. Second, conditioning on these possible

Table A.1: Odds of investor type distributions

Distribution	Probability		
$\phantom{aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa$	1/64		
5a + b	6/64		
4a + 2b	15/64		
3a + 3b	20/64		
2a + 4b	15/64		
a+5b	6/64		
6b	1/64		

aggregate outcomes, we derive the match probability of an investor initially at each broker in each of three scenarios: the "Stay" scenario $(j_l = l)$ where an investor stays at its initial broker at Stage 3, and the two alternative "Move" scenarios $(j_l \neq l)$ where an investor switches to one of the other brokers at Stage 3. Finally, we weigh these second-step results

by the probabilities of each first-step outcome to derive nine overall match probabilities, one for each initial broker-chosen broker pair lj.

We repeat this process for each of the five possible coalition structures $\mathcal{C} \in \mathcal{P}$. The resulting probabilities $P_j^e(\mathcal{C}, j_l, \mathbf{\Phi}_l^{Stay})$ are shown in Table A.2. With these probabilities in

Table A.2: Investor match probabilities

$\overline{\text{Coalition structure }\mathcal{C}}$	Match probabil	ity	for l in	nvestor	choos	ing broker j
{{1}, {2}, {3}}	_ l \	$\langle j \mid$	1	2	3	
	-	1	0.55	0.69	0.70	
	6	2	0.73	0.73	0.76	
		3	0.69	0.71	0.71	
{{1,2},{3}}	<i>l</i>	\bar{j}	1	2	3	
		1	0.56	0.69	0.70	
	4	2	0.75	0.75	0.70	
		3	0.70	0.73	0.69	
	<i>l</i>	\bar{j}	$\overline{1}$	2	3	
[[1 2] [2]]		1	0.61	0.69	0.70	
$\{\{1,3\},\{2\}\}$	4	2	0.74	0.66	0.77	
	•	3	0.70	0.69	0.73	
{{1}, {2, 3}}	<i>l</i>	\bar{j}	1	2	3	
		1	0.50	0.69	0.70	•
	4	2	0.66	0.74	0.77	
		3	0.66	0.71	0.71	
{{1,2,3}}	<i>l</i>	\bar{j}	1	2	3	
		1	0.55	0.69	0.70	-
	6	2	0.73	0.73	0.76	
		3	0.69	0.71	0.71	

hand, we can derive the "Stay Conditions" that must be satisfied if all investors are to remain at their initial brokers. Equation A.1.2 implies that each investor at initial broker l will choose to stay at broker l in Stage 3 if the investor's maximum utility from moving to a new broker j is no larger than the investor's maximum utility from moving to a new broker j:

$$\eta + \max\{(1 - f_{ll})P_l^e(\mathcal{C}, l, \mathbf{\Phi}_l^{Stay}), 0\} \ge \eta + \max_{j \ne l}\{(1 - f_{lj})P_j^e(\mathcal{C}, j, \mathbf{\Phi}_l^{Stay}), 0\} - \tau_{lj} \quad (A.1.3)$$

These conditions are shown explicitly by coalition structure in Table A.3.

Stage 2. Brokers set their fees f_j to maximize:

$$\max_{\mathbf{f}_j} \{ \pi_j^e(\mathcal{C}) = \mathbf{f}_j' \mathbf{n}_j^e(\mathcal{C}, \mathbf{f}) - \gamma \cdot n_{j,L}^e(\mathcal{C}, \mathbf{f}) - s_j \} . \tag{A.1.4}$$

Table A.3: Stay Conditions by coalition structure

Coalition structure \mathcal{C}	Stay Conditions
{{1},{2},{3}}	$(1 - f_{11}) \cdot 0.55 \ge \max\{(1 - f_{12}) \cdot 0.69, (1 - f_{13}) \cdot 0.70, 0\} - \tau$ $(1 - f_{22}) \cdot 0.73 \ge \max\{(1 - f_{21}) \cdot 0.73, (1 - f_{23}) \cdot 0.76, 0\} - \tau$ $(1 - f_{33}) \cdot 0.71 \ge \max\{(1 - f_{31}) \cdot 0.69, (1 - f_{32}) \cdot 0.71, 0\} - \tau$
{{1,2},{3}}	$(1 - f_{11}) \cdot 0.56 \ge \max\{(1 - f_{12}) \cdot 0.69, (1 - f_{13}) \cdot 0.70, 0\} - \tau$ $(1 - f_{22}) \cdot 0.75 \ge \max\{(1 - f_{21}) \cdot 0.75, (1 - f_{23}) \cdot 0.70, 0\} - \tau$ $(1 - f_{33}) \cdot 0.69 \ge \max\{(1 - f_{31}) \cdot 0.70, (1 - f_{32}) \cdot 0.73, 0\} - \tau$
{{1,3},{2}}	$ \begin{array}{l} (1 - f_{11}) \cdot 0.61 \geq \max\{(1 - f_{12}) \cdot 0.69, (1 - f_{13}) \cdot 0.70, 0\} - \tau \\ (1 - f_{22}) \cdot 0.66 \geq \max\{(1 - f_{21}) \cdot 0.74, (1 - f_{23}) \cdot 0.77, 0\} - \tau \\ (1 - f_{33}) \cdot 0.73 \geq \max\{(1 - f_{31}) \cdot 0.70, (1 - f_{32}) \cdot 0.69, 0\} - \tau \end{array} $
{{1}, {2,3}}	$(1 - f_{11}) \cdot 0.50 \ge \max\{(1 - f_{12}) \cdot 0.69, (1 - f_{13}) \cdot 0.70, 0\} - \tau$ $(1 - f_{22}) \cdot 0.74 \ge \max\{(1 - f_{21}) \cdot 0.66, (1 - f_{23}) \cdot 0.77, 0\} - \tau$ $(1 - f_{33}) \cdot 0.71 \ge \max\{(1 - f_{31}) \cdot 0.66, (1 - f_{32}) \cdot 0.71, 0\} - \tau$
{{1,2,3}}	$(1 - f_{11}) \cdot 0.55 \ge \max\{(1 - f_{12}) \cdot 0.69, (1 - f_{13}) \cdot 0.70, 0\} - \tau$ $(1 - f_{22}) \cdot 0.73 \ge \max\{(1 - f_{21}) \cdot 0.73, (1 - f_{23}) \cdot 0.76, 0\} - \tau$ $(1 - f_{33}) \cdot 0.71 \ge \max\{(1 - f_{31}) \cdot 0.69, (1 - f_{32}) \cdot 0.71, 0\} - \tau$

Since any tokenized market setup cost s_j is sunk by Stage 2, each broker's problem is to balance revenue per trade (which rises with the fees they set) against the number of trades that they expect to facilitate (which falls when fee increases induce investor movements away from the broker). For each coalition structure, we proceed in two steps. First, we derive the "best competing alternative" that is offered to an investor at l by one of their non-initial brokers $j \neq l$. In most cases this (Bertrand competition outcome) turns out to be a switcher-fee of $f_{lj} = 0$, reflecting that each broker is generally willing to acquire additional investors even at a zero fee because doing so increases the match probability of the broker's initial investors, which in turn raises the fee the broker can charge them. Second, we note that this best competing alternative defines the reservation utility that each broker must offer its initial investors in order to retain them. The best competing alternative thus pins down the right-hand side of each Stay Condition in Table A.3. Since brokers always charge the highest fee consistent with retaining their investors, they set fees such that each Stay Condition binds with equality, allowing us to rearrange each condition to derive the optimal stayer fee f_{ll} in each case.

No Coalition ({{1}, {2}, {3}}). First consider the fees that Brokers 1 and 2 charge their own initial investors—i.e., f_{11} and f_{22} . From the first two Stay Conditions we see that Broker 3 is the most attractive alternative broker for those investors when considering match probabilities alone. To ascertain whether Broker 3 would bid down to $f_{13} = 0$ and $f_{23} = 0$, consider the following three cases:

- (i) Both Brokers 1 and 2 charge the f_{11} and f_{22} that result from the Stay Conditions when $f_{13} = 0$ and $f_{23} = 0$;
- (ii) Both Brokers 1 and 2 charge fees above the level in case (i);

(iii) Only one of Broker 1 and 2, denoted as Broker X, charges fees above the level in case (i), while the other, denoted Broker Y, charges fees at the level in case (i).

In case (ii), Brokers 1 and 2 are charging fees above the level implied by the Stay Conditions, meaning Broker 3 could attract both brokers' initial investors by setting $f_{13} = 0$ and $f_{23} = 0$ —and doing so would be profitable for Broker 3, since attracting all investors implies clearing all trades intra-broker at zero marginal cost. Thus Brokers 1 and 2 do not charge the case (ii) fees in equilibrium. In case (iii), Broker 3 cannot undercut Broker X by charging $f_{X3} = 0$, since Broker Y maintains their initial investors and so Broker 3 could incur positive marginal costs when facilitating remaining legacy market transactions. Thus there exists a level of excess fees—i.e., fees above those in case (i)—that Broker X can charge at which it will not be undercut by Broker 3. This in turn implies that Broker 3's threat to charge $f_{Y3} = 0$ is not credible, since Broker 3 could incur positive marginal costs when facilitating legacy market transactions with Broker X. Thus Broker Y can also charge fees above the level in case (i), so case (iii) collapses into case (ii) and the only stable equilibrium is that which arises in case (i)—i.e., $f_{13} = 0$, $f_{23} = 0$ and

$$f_{11} = 1.83 \,\tau - 0.29 \tag{A.1.5}$$

$$f_{22} = 1.38 \tau - 0.04 . (A.1.6)$$

Lastly, we turn to f_{33} and consider the "best competing alternative" to Broker 3 that would be offered by Brokers 1 and 2. It can be shown that: (i) Broker 1 is willing to set $f_{31} = 0$, (ii) Broker 2 is not willing to set $f_{32} = 0$, but (iii) even so, Broker 2 is willing to set f_{32} sufficiently low that—when combined with the higher match probability that it offers relative to Broker 1—Broker 2 remains the best competing alternative. Specifically, Broker 2 is willing to lower its fee until $f_{32} = 0.02\gamma$, which when combined with the Stay Conditions for the No Coalition case implies that Broker 3 sets

$$f_{33} = 1.41\,\tau + 0.02\,\gamma \ . \tag{A.1.7}$$

1 & 2 Coalition ($\{\{1,2\},\{3\}\}$). If either Broker 1 or Broker 2 can attract Broker 3's initial investors, their marginal cost of processing transactions will always be zero since there will be no more OTC trades. Thus Brokers 1 and 2 are willing to bid down to $f_{31} = f_{32} = 0$, which when combined with Broker 3's Stay Condition gives

$$f_{33} = 1.45 \tau - 0.07 . \tag{A.1.8}$$

Next, it can be shown that the number of legacy market trades that Broker 3 expects to clear falls if it can attract Broker 1's initial investor, since Broker 3's greater likelihood of intra-broker matches outweighs the effect of hosting more investors who could require an inter-broker match. Thus Broker 3 always gains from acquiring Broker 1's investor, even if it earns zero fees from them directly, since the acquisition increases Broker 3's expected number of fee-paying intra-broker matches and lowers Broker 3's expenses. Thus, Broker 3 competes down to $f_{13} = 0$, which combines with Broker 1's Stay Condition to give

$$f_{11} = 1.78 \,\tau - 0.25 \;. \tag{A.1.9}$$

Lastly, we turn to Broker 2's initial investors. The second Stay Condition reveals that Broker 1 is the most competitive broker for these investors, other than Broker 2. It can further be shown that Broker 1's profits are higher in the "Move" case in which it sets $f_{21} = 0$ and attracts Broker 2's investors than in the "Stay" case in which it does not attract Broker 2's investors. Thus, Broker 1 and Broker 2 compete on fees down to the point where Broker 1 sets $f_{21} = 0$, which by the second Stay Condition implies that Broker 2 sets

$$f_{22} = 1.33 \,\tau \,. \tag{A.1.10}$$

1 & 3 Coalition ({{1,3},{2}}). First, an analogous argument to that from cases (i)-(iii) in the No Coalition scenario implies that $f_{13} = 0$ and $f_{23} = 0$ in equilibrium, and thus from the Stay Conditions we have:

$$f_{11} = 1.64 \,\tau - 0.15 \tag{A.1.11}$$

$$f_{22} = 1.52 \tau - 0.17. \tag{A.1.12}$$

Turning to f_{33} , we again consider the "best competing alternative" to remaining at Broker 3 for Broker 3's initial investors. From the Stay Conditions, we see that their match probability is higher at Broker 1 than at Broker 2. Calculating Broker 1's profits under the scenarios that (i) Broker 3's investors stay at Broker 3 and (ii) Broker 1 attracts Broker 3's investors by setting $f_{31} = 0$, we find that Broker 1 prefers the latter. Thus Broker 1 is willing to compete with Broker 3 on fees down to $f_{31} = 0$, so Broker 3's fees are determined by the corresponding Stay Condition, giving

$$f_{33} = 1.36 \,\tau + 0.04 \,. \tag{A.1.13}$$

2 & 3 Coalition ({{2,3},{1}}). Again, an analogous argument to that from cases (i)-(iii) in the No Coalition scenario implies that $f_{13} = 0$ and $f_{23} = 0$ in equilibrium, and thus from the Stay Conditions we have:

$$f_{11} = 2.00 \,\tau - 0.41 \tag{A.1.14}$$

$$f_{22} = 1.35 \tau - 0.04 . \tag{A.1.15}$$

Turning to f_{33} , from the match probabilities we see that Broker 2 is again the best competing alternative to Broker 3 for Broker 3's initial investors. Furthermore, it can be shown that Broker 2 is willing to compete down to $f_{32} = 0$, again implying a Bertrand fee setting game with Broker 3—which in turn implies that Broker 3's equilibrium fees are given by its Stay Condition, producing:

$$f_{33} = 1.40\,\tau$$
 (A.1.16)

Grand Coalition ($\{\{1,2,3\}\}$). Since all brokers are part of the tokenized platform, the legacy technology is never used and the marginal cost of processing transactions is always zero. Thus non-initial brokers are always willing to bid down to $f_{lj} = 0$ (for all $l \neq j$), and a Bertrand race between them and the initial broker l ensures that this occurs in equilibrium.

Thus the Stay Conditions become:

$$(1 - f_{11}) \cdot 0.55 = \max\{0.69, 0.70, 0\} - \tau \tag{A.1.17}$$

$$(1 - f_{22}) \cdot 0.73 = \max\{0.73, 0.76, 0\} - \tau \tag{A.1.18}$$

$$(1 - f_{33}) \cdot 0.71 = \max\{0.69, 0.71, 0\} - \tau \tag{A.1.19}$$

which rearrange to give

$$f_{11} = 1.83 \,\tau - 0.29 \tag{A.1.20}$$

$$f_{22} = 1.38\,\tau - 0.04\tag{A.1.21}$$

$$f_{33} = 1.41 \,\tau \,. \tag{A.1.22}$$

Summary. We have shown that for each coalition structure there exists a unique set of fees charged such that both (i) all brokers set fees to maximize expected profits and (ii) all investors maximize expected utility by choosing to stay, under the prior that all other investors will stay. Thus, for any coalition structure \mathcal{C} that emerges from Stage 1, there is a unique equilibrium of the resulting subgame. Since all brokers foresee this equilibrium, and their corresponding expected profits $\pi_j^e(\mathcal{C})$, these are the payoffs that they consider when negotiating in Stage 1—to which we now turn.

Stage 1. We first calculate each broker's expected profits from each coalition structure, shown in Table A.4. The higher is s, the more attractive is No Coalition relative to the other coalition structures, since brokers' payoffs in that case (shown in the top row of the table) do not depend on s, whereas higher s reduces the payoff of all the brokers forming a tokenized market in each of the other coalition structures. The higher is γ , the more attractive is the Grand Coalition relative to the other coalition structures, since brokers' payoffs in that case (shown in the bottom row of the table) do not depend on γ , whereas a higher γ reduces each broker's payoff under all other coalition structures.

Table A.4: Brokers' expected profits by coalition structure

Coalition structure $\mathcal C$	$\pi_1^e(\mathcal{C})$	$\pi_2^e(\mathcal{C})$	$\pi_3^e(\mathcal{C})$	$\Pi^e(\mathcal{C})$
{{1}, {2}, {3}}	$\tau - 0.55\gamma - 0.16$	$2\tau - 0.45\gamma - 0.06$	$3\tau - 0.58\gamma$	$6\tau - 1.58\gamma - 0.22$
{{1,2},{3}}	$\tau - 0.31\gamma - 0.50 s - 0.14$	$2\tau - 0.25\gamma - 0.50 s$	$3\tau - 0.56\gamma - 0.14$	$6\tau - s - 1.12\gamma - 0.28$
{{1,3},{2}}	$\tau - 0.11\gamma - 0.50 s - 0.09$	$2\tau - 0.31\gamma - 0.23$	$3\tau - 0.20\gamma - 0.50 s + 0.09$	$6\tau - s - 0.62\gamma - 0.23$
{{1}, {2, 3}}	$\tau - 0.50\gamma - 0.20$	$2\tau - 0.17\gamma - 0.50 s - 0.06$	$3\tau - 0.33\gamma - 0.50 s$	$6\tau - s - 1.00\gamma - 0.26$
$\{\{1,2,3\}\}$	$\tau - 0.33 s - 0.16$	$2\tau - 0.33 s - 0.06$	$3\tau - 0.33s$	$6\tau - s - 0.22$

We proceed through the iterated deletion of dominated coalition structures. First, we consider the 1 & 2 Coalition. Comparing the second and third rows of Table A.4 reveals that Broker 1 always prefers the 1 & 3 Coalition to the 1 & 2 Coalition, for any values of

the parameters. Broker 3 also prefers the 1 & 3 Coalition unless:

$$\pi_3^e(1 \& 3) = 3\tau - 0.20\gamma - 0.50 s + 0.09 < \pi_3^e(1 \& 2) = 3\tau - 0.56\gamma - 0.14$$

$$\iff s > 0.72\gamma + 0.46 . \tag{A.1.23}$$

However, if this condition does hold, then we can also see (by substituting it into $\pi_1^e(1 \& 2)$ in Table A.4) that Broker 1 would not choose to form the 1 & 2 Coalition in the first place, since under that condition they strictly prefer No Coalition. Thus the 1 & 2 Coalition never arises in equilibrium: either Brokers 1 and 3 prefer to instead form the 1 & 3 Coalition, or Broker 1 prefers to not form any coalition.

Second, we consider the 2 & 3 Coalition. Comparing the third and fourth rows of Table A.4 reveals that Broker 3 always prefers the 1 & 3 Coalition to the 2 & 3 Coalition. Broker 1 also prefers the 1 & 3 Coalition unless:

$$\pi_1^e(1 \& 3) = \tau - 0.11\gamma - 0.50 s - 0.09 < \pi_1^e(2 \& 3) = \tau - 0.50\gamma - 0.20$$

$$\iff s > 0.78\gamma + 0.22 . \tag{A.1.24}$$

However, if this condition does hold, then we can see (by substituting it into $\pi_3^e(2 \& 3)$ in Table A.4) that Broker 3 would not choose to form the 2 & 3 Coalition in the first place, since under that condition they strictly prefer No Coalition. Thus the 2 & 3 Coalition also never arises in equilibrium: either Brokers 1 and 3 prefer to instead form the 1 & 3 Coalition, or Broker 3 prefers to not form any coalition.

Next we turn to the Grand Coalition. The unique attraction of this outcome for the brokers is its elimination of the legacy transaction cost γ . Thus, the Grand Coalition is most attractive relative to the remaining coalition structures when this cost is maximized, i.e., when $\gamma = \frac{1}{6}$. In this case, Brokers 1 and 3 would nonetheless prefer to deviate to form the 1 & 3 Coalition unless:

$$\pi_1^e(1 \& 3) = \tau - 0.11 * \left(\frac{1}{6}\right) - 0.50 s - 0.09 < \pi_1^e(Grand) = \tau - 0.33 s - 0.16$$
 $\iff s > 0.30$
(A.1.25)

i.e., setup costs are so high that Broker 1 prefers to share them through the Grand Coalition.³⁷ However, even in this case the Grand Coalition would not form, since when $\gamma = \frac{1}{6}$ we have $\pi_1^e(\text{No Coalition}) = \tau - 0.25$ yet when s > 0.30 we have $\pi_1^e(\text{Grand}) < \tau - 0.26$. Thus at least one Broker always prefers to deviate from the Grand Coalition, so it never forms in equilibrium. Either the cost s of forming a tokenized market is low enough that Brokers 1 and 3 prefer to form their own partial coalition, or s is so high that at least one broker prefers not to form any coalition.

Having established that only No Coalition and the 1 & 3 Coalition can form in equilib-

$$\pi_3^e(1 \& 3) = 3\tau - 0.20\gamma - 0.50 s + 0.09 > \pi_3^e(Grand) = 3\tau - 0.33 s$$

which for $\gamma = \frac{1}{6}$ yields s < 0.33, i.e., Broker 1 is the marginal broker.

 $^{^{37}}$ We note that whenever s is sufficiently low that Broker 1 prefers to join form the 1 & 3 Coalition, so does Broker 3, since Broker 3 prefers the 1 & 3 Coalition whenever

rium, we identify the parameter ranges for which each occur. Comparing the first and third rows of the Table A.4, we see that Broker 3 prefers the 1 & 3 Coalition over No Coalition when

$$\pi_3^e(1 \& 3) = 3\tau - 0.20\gamma - 0.50 s + 0.09 > \pi_3^e(\text{No Coalition}) = 3\tau - 0.58\gamma$$
 $\iff s < 0.18 + 0.76\gamma$
(A.1.26)

and that Broker 1 prefers the 1 & 3 Coalition over No Coalition when

$$\pi_1^e(1 \& 3) = \tau - 0.11\gamma - 0.50 s - 0.09 > \pi_1^e(\text{No Coalition}) = \tau - 0.55\gamma - 0.16$$

 $\iff s < 0.14 + 0.88\gamma$.

or, as expressed in exact fractions:

$$s < \frac{1}{8} + \frac{7}{8}\gamma$$
 (A.1.27)

The identity of the marginal broker depends on which of the two terms on the right-hand side dominates. At the upper bound of $\gamma = \frac{1}{6}$, Broker 3's threshold level of s is 0.31, whereas Broker 1's threshold level of s is 0.29. Thus, Broker 1's threshold is always below that of Broker 3, so Broker 1's condition (i.e., equation A.1.27) forms the boundary. When s is below $\frac{1}{8} + \frac{7}{8}\gamma$ the 1 & 3 Coalition forms, while when s is above that level No Coalition forms. When $s = \frac{1}{8} + \frac{7}{8}\gamma$, Broker 1 is indifferent and and so the 1 & 3 Coalition forms as described in footnote 24.

A.2 Proof of Lemma 2

Expected aggregate welfare $W^e(\mathcal{C})$ for each coalition structure is the sum of four terms. First is the sum of investors' endowments 6η : these are either consumed or paid to investors in fees. Second is the total expected gains from trade. This can be calculated as 4.125, which is the maximum number of matches that can take place for each distribution in Table A.1, multiplied by the probability of that distribution occurring. This number is constant across coalition structures, since the coalition structure only affects the location of the matches, not their total quantity. The third term is the cost of creating a tokenized market, s, which is incurred under all coalition structures except No Coalition. The final term is the expected total cost $\gamma \cdot n_L^e$ of processing legacy market trades, where $n_L^e = n_{1,L}^e + n_{2,L}^e + n_{3,L}^e$ is the total number of such trades expected to be facilitated across all three brokers.³⁸

Table A.5 summarizes the resulting welfare expressions for each coalition structure. It is immediately apparent that no partial coalition structure ever maximizes welfare: in all such cases the full setup cost s is incurred but some (costly) trades are still expected to occur on the legacy market. Thus expected welfare is maximized by either No Coalition, the Grand Coalition, or both. It can be shown that $n_L^e(\text{No Coalition}) = \frac{13}{8}$, so the set of points for

³⁸Note that the cost of switching brokers, τ , does not appear in expected aggregate welfare because it is never incurred in the Stay Equilibrium.

which welfare is equal under the No Coalition and the Grand Coalition is given by:

$$W^{e}(\text{No Coalition}) = 6\eta + 4.125 - \gamma \cdot n_{L}^{e}(\text{No Coalition}) = W^{e}(\text{Grand}) = 6\eta + 4.125 - s$$

$$\iff s = \frac{13}{8}\gamma. \tag{A.2.1}$$

Thus, when $s > \frac{13}{8}\gamma$ welfare is maximized by No Coalition, when $s < \frac{13}{8}\gamma$ welfare is maximized by the Grand Coalition, and when $s = \frac{13}{8}\gamma$ welfare is equal in under both coalition structures.

Table A.5: Welfare by coalition structure

Coalition structure $\mathcal C$	Expected aggregate welfare $W^e(\mathcal{C})$
{{1}, {2}, {3}}	$6\eta + 4.125 - \gamma \cdot n_L^e$ (No Coalition)
{{1,2},{3}}	$6\eta + 4.125 - s - \gamma \cdot n_L^e(1 \& 2)$
{{1,3},{2}}	$6\eta + 4.125 - s - \gamma \cdot n_L^e(1 \& 3)$
{{1}, {2,3}}	$6\eta + 4.125 - s - \gamma \cdot n_L^e(2 \& 3)$
$\{\{1,2,3\}\}$	$6\eta + 4.125 - s$

A.3 Proof of Lemma 3

The proof proceeds in two stages. First we solve the game when the interoperability mandate is imposed, then we relate the results to the outcomes of the baseline model.

A.3.1 Equilibrium under the interoperability mandate

Stages 3 and 2 of the game are identical to those described in the proof of Lemma 1. Thus, brokers' payoffs from coalition structure C, if it emerges from the new (three-sub-stage) Stage 1, are the same as those given in Table A.4. We thus proceed by backward induction through the Stage 1 sub-stages.

Stage 1.3. This stage only exists if a partial coalition formed at Stage 1.2. We consider each of the three possible cases.

1 & 2 Coalition. If the 1 & 2 Coalition formed at Stage 1.2, Broker 3 faces the following two options. If it does not join the tokenized platform its expected profits are

$$\pi_3^e (1 \& 2 \text{ Coalition}) = 3\tau - 0.56\gamma - 0.14$$
 (A.3.1)

as in Table A.4. If instead it chooses to accept the offer to join the coalition at no cost, its profit is the Grand Coalition profit in Table A.4, minus any share of the setup cost—giving π_3^e (Grand via 1 & 2) = 3τ . This dominates π_3^e (1 & 2 Coalition), so Broker 3 chooses to join.

1 & 3 Coalition. By analogous reasoning, in this case Broker 2 faces a choice between

$$\pi_2^e (1 \& 3 \text{ Coalition}) = 2\tau - 0.31\gamma - 0.23$$
 (A.3.2)

and $\pi_2^e(Grand via 1 \& 3) = 2\tau - 0.06$. Again, the latter dominates so Broker 2 chooses to join the coalition.

2 & 3 Coalition. Finally, by the same logic Broker 1 chooses between

$$\pi_1^e(2 \& 3 \text{ Coalition}) = \tau - 0.50\gamma - 0.20$$
 (A.3.3)

and $\pi_1^e(Grand \text{ via } 2 \& 3) = \tau - 0.16$. Again, the latter dominates so Broker 1 chooses to join.

Stage 1.2. Foreseeing that any excluded broker will subsequently accept the offer to join, brokers consider the resulting expected profit structure shown in Table A.6. The first and fifth rows are unchanged relative to the baseline case without interoperability (Table A.4), while the middle rows reflect that partial coalitions all ultimately result in the Grand Coalition forming, simply with different distributions of the setup cost s. To proceed, we now consider the coalition structures that result from negotiation at Stage 1.2 for various different investment budget limits s_i^{max} set by the brokers at Stage 1.

All brokers unconstrained. Let $s_1^{max} \geq 0.5$, $s_1^{max} \geq 0.5$ and $s_1^{max} \geq 0.5$. In this case, no broker is constrained in the amount that it can invest in setup costs, so all rows in Table A.6 are feasible. To see the implications, label the three brokers X, Y and Z (in any order) and consider the perspective of Broker X. For any coalition structure not containing the coalition $\{X\}$, they will prefer the alternative coalition structure $\{\{X\}, \{Y, Z\}\}$ in which the other brokers form a partial coalition, which X then joins at Stage 1.3. Thus the only possible equilibria are the coalition structures that contain $\{X\}$ —i.e., $\{\{X\}, \{Y, Z\}\}$ and $\{\{X\}, \{Y\}, \{Z\}\}$. But since Y and Z reason in the same way, $\{\{X\}, \{Y, Z\}\}$ can never form, so the only possible equilibrium is $\{\{X\}, \{Y\}, \{Z\}\}$.

Two brokers unconstrained. Consider now the case in which any two brokers, Y and Z, have $s_Y^{max} \geq 0.5$ and $s_Z^{max} \geq 0.5$, while the third broker X has $s_X^{max} < 0.5$. Again, for any coalition structure not containing the coalition $\{X\}$, X will prefer the alternative coalition structure $\{\{X\}, \{Y, Z\}\}$ in which the other brokers form a partial coalition, which X then joins at Stage 1.3. For Y and Z, the hope of free-riding in this way is not available, since in each case the other two brokers lack sufficient resources to meet the investment cost s alone. Thus the only possible equilibria are the coalition structures that contain $\{X\}$ —i.e., $\{\{X\}, \{Y, Z\}\}$ and $\{\{X\}, \{Y\}, \{Z\}\}$.

One or zero brokers unconstrained. If fewer than two brokers have $s_j^{max} \geq 0.5$, free-riding is impossible for all brokers. Given that investment costs are shared equally among coalition members, there are two possible scenarios: (i) if one or more brokers have $s_j^{max} < 0.33$, only No Coalition is possible; (ii) if instead all three brokers have $s_j^{max} \geq 0.33$, the Grand Coalition is also possible. Specifically, the Grand Coalition will form if:

$$\pi_1^e(Grand) = \tau - 0.33 s - 0.16 \ge \pi_1^e(No \text{ Coalition}) = \tau - 0.55\gamma - 0.16$$

$$\iff s \le 1.67\gamma \tag{A.3.4}$$

and

$$\pi_2^e(\text{Grand}) = 2\tau - 0.33 \, s - 0.06 \ge \pi_2^e(\text{No Coalition}) = 2\tau - 0.45\gamma - 0.06$$

$$\iff s \le 1.36\gamma$$
in fractions: $s \le \frac{27}{20}\gamma$ (A.3.5)

and

$$\pi_3^e(Grand) = 3\tau - 0.33 s \ge \pi_3^e(No \text{ Coalition}) = 3\tau - 0.58\gamma$$

$$\iff s \le 1.76\gamma, \tag{A.3.6}$$

which reduces to the single condition $s \leq \frac{27}{20}\gamma$. Thus Broker 2 is the marginal broker, for whom setup costs must fall the most before the Grand Coalition is preferable to No Coalition.

Table A.6: Brokers' expected profits under interoperability mandate

Coalition structure at Stage 1.2	$\pi_1^e(\mathcal{C})$	$\pi_2^e(\mathcal{C})$	$\pi_3^e(\mathcal{C})$
{{1},{2},{3}}	$\tau - 0.55\gamma - 0.16$	$2\tau - 0.45\gamma - 0.06$	$3\tau - 0.58\gamma$
{{1,2},{3}}	$\tau - 0.50 s - 0.16$	$2\tau - 0.50 s - 0.06$	3τ
{{1,3},{2}}	$\tau - 0.50 s - 0.16$	$2\tau - 0.06$	$3\tau - 0.50s$
{{1}, {2, 3}}	$\tau - 0.16$	$2\tau - 0.50 s - 0.06$	$3\tau - 0.50s$
{{1,2,3}}	$\tau - 0.33 s - 0.16$	$2\tau - 0.33 s - 0.06$	$3\tau - 0.33s$

Stage 1.1. Anticipating the outcomes above, at Stage 1.1 each Broker X foresees that:

1. If they set $s_X^{max} \ge 0.5$, the outcome after Stage 1.3 will be either No Coalition or a Grand Coalition for which X pays $s_X = 0.5$ —i.e., more than an equal share;

- 2. If they set $s_X^{max} < 0.33$, the outcome after Stage 1.3 will be No Coalition or a Grand Coalition for which X pays $s_x = 0$;
- 3. If they set $0.33 \le s_X^{max} < 0.5$, there are three possibilities: No Coalition, a Grand Coalition to which X contributes equally $(s_X = 0.33)$, or a Grand Coalition to which X contributes nothing $(s_X = 0)$.

Policy 1 is dominated by the other two, so X will never choose it. All brokers reason the same way, which in turn rules out the Grand Coalition outcome under Policy 2—since if no broker sets $s_j^{max} \geq 0.5$, there is never an opportunity to free-ride. Thus Policy 2 always produces No Coalition. Similarly, since no broker sets $s_j^{max} \geq 0.5$, the free-riding Grand Coalition outcome under Policy 3 is also ruled out, so Policy 3 always produces either No Coalition or the Grand Coalition to which all brokers contribute equally (i.e., the 'Egalitarian Grand Coalition').

In summary, broker j's choices at Stage 1.1 reduce to a choice between allowing the possibility of the Egalitarian Grand Coalition at Stage 1.2 (which requires setting $0.33 \le s_X^{max} < 0.5$) and ruling it out (by setting $s_X^{max} < 0.33$). Broker 1 does the former if condition A.3.4 holds; Broker 2 does likewise if condition A.3.5 holds; and Broker 3 does likewise if condition A.3.6 holds. If condition A.3.5 holds, so must the other two, so the outcome depends only on condition A.3.5—i.e., on whether Broker 2 is willing to form the Egalitarian Grand Coalition. Thus under the interoperability mandate, the Egalitarian Grand Coalition forms if $s \le \frac{27}{20}\gamma$, and No Coalition forms otherwise. This outcome is summarized by the green boundary in Figure 4a.

A.3.2 Relation to the outcomes of the baseline model

The result in Lemma 3 follows immediately from comparing the outcome derived in the previous subsection to the excessive investment region presented in Proposition 1. Graphically, comparing Figures 3b and 4a reveals that the excessive investment region in the former (where the 1 & 3 Coalition forms yet No Coalition is optimal) changes to the optimal No Coalition outcome in the latter.

A.4 Proof of Lemma 4

The result follows immediately from comparing the outcome derived in Section A.3.1 to the insufficient tokenization region presented in Proposition 1. Graphically, comparing Figures 3b and 4a reveals that the lower part of the insufficient tokenization region in the former changes to the optimal Grand Coalition outcome in the latter. The boundary of this lower part is given by the condition $s \leq \frac{27}{20} \gamma$ derived in Section A.3.1.

A.5 Proof of Lemma 5

The result follows immediately from comparing the outcome derived in Section A.3.1 to the insufficient tokenization region presented in Proposition 1. Graphically, comparing Figures 3b and 4a reveals that, in the upper part of the insufficient tokenization region in the former, No Coalition results in the latter. The upper boundary of this part is given by the condition

 $s \leq \frac{13}{8}\gamma$ in Proposition 1. The lower boundary of this part is given by the condition $s > \frac{27}{20}\gamma$ derived in Section A.3.1.

A.6 Proof of Lemma 6

Let the policymaker bear σ of the total setup cost s when $\frac{27}{20}\gamma < s \leq \frac{13}{8}\gamma$. Following Section A.3.1 above, Broker 2 then chooses the Grand Coalition over No Coalition if:

$$s \le \frac{27}{20}\gamma + \sigma \ . \tag{A.6.1}$$

From Lemma 2, welfare is maximized when the corresponding boundary is:

$$s \le \frac{13}{8}\gamma \ . \tag{A.6.2}$$

These two conditions are thus equivalent when:

$$\frac{27}{20}\gamma + \sigma = \frac{13}{8}\gamma$$

$$\iff \sigma = \frac{11}{40}\gamma.$$
(A.6.3)

Thus, a subsidy of this level or greater prevents the insufficient tokenization that would otherwise result, by tipping Broker 2 into choosing the Grand Coalition.

A.7 Proof of Proposition 3

Stages 3 and 2 are identical to those in the baseline setup. We derive the equilibrium outcomes at Stage 1 in three steps.

First, we note that when side-payments are possible any equilibrium outcome (C, \mathbf{v}) must maximize expected aggregate broker profits. The proof of this statement is by contradiction. Suppose that an equilibrium outcome (C, \mathbf{v}) exists that does not maximize expected joint broker profits Π^e . By definition, there exists some alternative coalition structure $C' \neq C$ such that (C', \mathbf{v}) does maximize expected joint broker profits. Since total expected profits are larger, there must then exist some trio of net transfers \mathbf{v}' such that $\pi_j^e(C', \mathbf{v}', s, \gamma) > \pi_j^e(C, \mathbf{v}, s, \gamma) \, \forall \, j$ (i.e., all brokers are strictly better off). Thus there does exist an alternative pair (C', \mathbf{v}') containing at least one new coalition $C' \notin C$ such that all brokers in C' are strictly better off than under (C, \mathbf{v}) —and the required side-payments will be acceptable to the payers, since all brokers are better off—so (C, \mathbf{v}) cannot be an equilibrium.

Second, we note that expected aggregate broker profits are maximized by: (i) No Coalition when $s \geq \frac{7313}{4640}\gamma$, and (ii) the Grand Coalition when $s \leq \frac{7313}{4640}\gamma$. To see this, we first note that at Stage 1 brokers' expected aggregate profits by coalition structure are the same as those shown in the final column of Table A.4 above, since total side-payments must sum to zero. That column reveals that aggregate profits under the Grand Coalition are always greater than those under any partial coalition. Comparing $\Pi^e(Grand)$ to $\Pi^e(No Coalition)$ then reveals that aggregate broker profits are maximized by the Grand Coalition when

 $s \le 1.58\gamma = \frac{7313}{4640}\gamma$ and by No Coalition when $s \ge 1.58\gamma = \frac{7313}{4640}\gamma$.

Combining these two steps and applying the tie-breaking assumption described in Section 2.5 gives that, when brokers can make side-payments, for any equilibrium outcome (C, \mathbf{v}) the coalition structure C must be: (i) No Coalition when $s > \frac{7313}{4640}\gamma$; and (ii) the Grand Coalition when $s \leq \frac{7313}{4640}\gamma$. The proposition then follows from comparing this result to Lemma 2. When $\frac{7313}{4640}\gamma < s < \frac{13}{8}\gamma$, the Grand Coalition alone is socially optimal but a private equilibrium must feature No Coalition, implying insufficient tokenization and underinvestment.

A.8 Proof of Proposition 4

The result follows from the proof of Proposition 3, and the discussion of a tokenization subsidy in Section 5. A subsidy σ induces brokers at point (γ, s) to make the decision that they would otherwise make at point $(\gamma, s-\sigma)$. The proof of Proposition 3 shows that the fully flexible side-payments equilibrium fails to maximize welfare only when brokers' perceived s is too high, such that an equilibrium features No Coalition when the Grand Coalition is socially optimal. It also shows that when brokers' perceived s is sufficiently low, equilibrium must feature the Grand Coalition. Therefore, a sufficiently large subsidy—lowering brokers' perceived s to the point where equilibrium features the Grand Coalition—can always align equilibrium outcomes with social welfare.

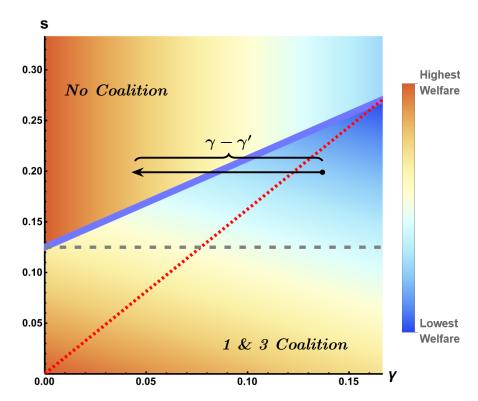
A.9 Proof of Proposition 5

Stages 2 and 3 are identical to those in the baseline setup. At the start of Stage 1, each broker must choose whether to join the ILI. A given Broker X reasons as follows. First, joining the ILI costs nothing and potentially lowers the cost to Broker X of some or all of its residual trades (i.e., those trades not cleared on the intra-broker or tokenized markets). Second, not joining the ILI could—if the other two brokers both join the ILI—raise the relative cost to those brokers of trading with Broker X instead of each other. In such a case, the other two brokers would choose to clear any residual trades among themselves before doing so with Broker X, reducing the match probability that Broker X could offer its investors and hence reducing the fees that Broker X could charge in equilibrium. On both considerations, Broker X benefits from joining the ILI. All brokers reason the same, and so all join the ILI at the start of Stage 1. Thus the remainder of Stage 1 proceeds as described in Section A.1.2, except with γ' replacing γ in all cases. Equilibrium coalition structures are then the same as those in the baseline, as shown by the blue line in Figure A.1, except with costs given by γ' instead of γ . Figure A.1 then reveals that: (i) for any point (γ, s) with $s > \frac{1}{8}$ —i.e., for any point above the gray dashed line—there exists a point (γ', s) with $\gamma' < \gamma$ at which a No Coalition outcome results in equilibrium and is socially optimal (the arrow in Figure A.1 indicates one such example); and (ii) no such point exists when $s \leq \frac{1}{8}$ —instead, the 1 & 3 Coalition results in equilibrium and is not socially optimal.

A.10 Proof of Proposition 6

We provide a visual proof. Within Figure A.1, for any arbitrarily efficient ILI (defined by $0 < \gamma' < \gamma$), one can define a maximum level of the setup cost s' > 0 such that the rectangle

Figure A.1: Equilibrium with independent ledger infrastructure



R between the origin and the point (γ', s') is identical to the corresponding rectangle in Figure 3b. Within R, both excessive investment and insufficient tokenization must occur in equilibrium—as can be seen by considering any rectangle of positive side length whose bottom-left corner is at the origin in Figure 3b. Thus all three policy zones in Figure 4b are also contained within R, so the same analysis as in Proposition 2 applies; when used alone neither an interoperability mandate nor public-private cost-sharing can always achieve the socially optimal outcome, but a combination of the two can.

